SYMPLECTIC DYNAMIC SYSTEMS ON TIME SCALES: QUADRATIC FUNCTIONALS AND OSCILLATION THEORY

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There exists parallel oscillation theories of Sturm-Liouville differential and difference equations

$$(r(t)x')' + p(t)x = 0 \quad \text{and} \quad \Delta(r_k \Delta x_k) + p_k x_{k+1} = 0$$

which can be unified using the concept of *time scale* equations. Recall that a time scale $\mathbb{T}$ is any closed subset of the real line $\mathbb{R}$. For a function f defined on $\mathbb{T}$, the operator of generalized derivative is defined which reduces to the ususal derivative in case $\mathbb{T} = \mathbb{R}$ and to the usual forward difference in case $\mathbb{T} = \mathbb{Z}$. In a dynamic equation on $\mathbb{T}$ this operator plays the same role as the usual derivative in differential equations and the forward difference in difference equations.

We will offer a similar unified approach to linear Hamiltonian differential systems

$$x' = A(t)x + B(t)u, \quad u' = C(t)x - A^T(t)u,$$

where A, B, C are $n \times n$ matrices and B, C are symmetric, and their difference counterparts – symplectic difference systems

$$z_{k+1} = S_k z_k,$$

where $z = \binom{x}{u}$, S is $2n \times 2n$ symplectic matrix, i.e.,

$$\mathcal{J}^T S \mathcal{J} = S \quad \text{and} \quad \mathcal{J} = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}.$$

The presented results were achieved in joint investigation with R. Hilscher, Masaryk University, Brno.