

MISSOURI S&T MISSOURI UNIVERSITY OF SCIENCE AND TECHNOLOGY

Chapter 3

Discrete Random Variables

- 3.1 Random Variables
- 3.2 Probability Distributions
- 3.3 Expected Values
- 3.4 The Binomial Probability Distribution
- 3.5 Hypergeometric and Negative Binomial Distributions**
- 3.6 The Poisson Probability Distribution

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Hypergeometric Experiment

1. The population to be sampled consists of **N** elements
2. Each element can be characterized as success or failure, and there are **M** successes in the population
3. A sample of **n** elements is selected without replacement in such a way that each subset of size **n** is equally likely to be chosen

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Example

A box contains 20 batteries. Of those, 4 batteries are broken. Select 3 batteries from the box (without replacement). Let **X** be the number of broken batteries among the 3 selected.

Find the pmf of X.

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Hypergeometric RV

The **hypergeometric random variable X** associated with a hypergeometric experiment is defined as the number of successes in the experiment. Its pmf is given by

$$p(x) = h(x; n, M, N) = \frac{\binom{M}{x} \binom{N-M}{n-x}}{\binom{N}{n}}$$

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Example

Five individuals from an animal population have been caught, tagged, and released to mix into the population. After a certain time, a random sample of 10 of these animals is selected. If there are 25 animals of this type in the region, find the probability that

- Exactly two animals are tagged in the second sample
- At most two animals are tagged in the second sample.

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Expectation and Variance of the Hypergeometric RV

If **X** is a hypergeometric rv with parameters **N**, **M** and **n**, then

$$E(X) = np \text{ with } p = M/N$$

$$V(X) = np(1-p)(N-n)/(N-1)$$

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Example

Five individuals from an animal population have been caught, tagged, and released to mix into the population. After a certain time, a random sample of 10 of these animals is selected. Suppose there are 25 animals of this type in the region. Let X be the number of tagged animals in the second sample.

- Find $E(X)$
- Find $V(X)$

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Negative Binomial Experiment

1. The experiment consists of a sequence of independent trials
2. Each trial can result in success or failure
3. The probability of success p is constant from trial to trial
4. The experiment continues until a total of r successes have been observed

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Example

A box contains 20 bulbs. Of those, 4 bulbs are broken. Select bulbs from the box (with replacement) until 3 broken bulbs are found. Let X be the number of working bulbs that are picked before the third broken bulb is found.

Find the pmf of X .

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Negative Binomial RV

The **negative binomial random variable X** associated with a negative binomial experiment is defined as the number of failures in the experiment preceding the r th success. It's pmf is given by

$$p(x) = nb(x; r, p) = \binom{x+r-1}{r-1} p^r (1-p)^x$$

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Example

A newspaper wants to interview five people in a city about a certain matter. Assume that the probability that a randomly selected person agrees to the interview is 0.2. Find the probability that

- 15 people must be asked before 5 are found who agree to participate
- at most 15 people must be asked before 5 are found who agree to participate.

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Expectation and Variance of the Negative Binomial RV

If X is a negative binomial rv with parameters r and p , then

$$E(X) = r(1-p)/p$$

$$V(X) = r(1-p)/p^2$$

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Example

A newspaper wants to interview five people in a city about a certain matter. Assume that the probability that a randomly selected person agrees to the interview is 0.2. Let X be the number of people who do not agree to be interviewed before the fifth person agrees.

- Find $E(X)$
- Find $V(X)$

