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## Chapter 3

# Discrete Random Variables

- 3.1 Random Variables
- 3.2 Probability Distributions
- 3.3 Expected Values
- 3.4 The Binomial Probability Distribution
- 3.5 Hypergeometric and Negative Binomial Distributions
- 3.6 The Poisson Probability Distribution**

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## Poisson RV

The **Poisson random variable**  $X$  with parameter  $\lambda > 0$  is the rv with the pmf

$$p(x) = p(x; \lambda) = \frac{e^{-\lambda} \lambda^x}{x!}$$

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## Example

Suppose the number of phone calls arriving within one minute at a call center is a Poisson rv with parameter  $\lambda=5$ . Find the probability that

- no call arrives within the next minute
- two calls arrive within the next minute
- at most two calls arrive within the next minute
- at most seven calls arrive within the next minute.

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## Expectation and Variance of the Poisson RV

If  $X$  is a Poisson rv with parameter  $\lambda$ , then

$$E(X) = \lambda$$

$$V(X) = \lambda$$

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## Example

Suppose the number of phone calls arriving within one minute at a call center is a Poisson rv with expected value 6. Find the probability that

- no call arrives within the next minute
- six calls arrive within the next minute.

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## Poisson RV as a Limit

Suppose that in the binomial pmf  $b(x;n,p)$  we let  $n \rightarrow \infty$  and  $p \rightarrow 0$  in such a way that  $np$  approaches a value  $\lambda > 0$ . Then

$$b(x;n,p) \rightarrow p(x;\lambda)$$

As a rule of thumb, this approximation can safely be applied if  $n > 50$  and  $np < 5$ .

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## Example

Assume that the probability of any given page (of a certain publisher's book) containing at least one typographical error is 0.005 and errors are independent from page to page. Find the probability that one of this publisher's 400-page book will contain

- **exactly one page with errors**
- **at most three pages with errors.**

