Chapter 3

Discrete Random Variables

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Poisson RV

The Poisson random variable X with parameter λ>0 is the rv with the pmf

$$p(x) = p(x;\lambda) = \frac{e^{-\lambda}\lambda^{x}}{x!}$$

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Example

Suppose the number of phone calls arriving within one minute at a call center is a Poisson rv with parameter λ=5. Find the probability that

- **no call arrives within the next minute**
- **two calls arrive within the next minute**
- **at most two calls arrive within the next minute**
- **at most seven calls arrive within the next minute.**

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Expectation and Variance of the Poisson RV

If X is a Poisson rv with parameter λ, then

E(X)=λ

V(X)=λ

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Example

Suppose the number of phone calls arriving within one minute at a call center is a Poisson rv with expected value 6. Find the probability that

- **no call arrives within the next minute**
- **six calls arrive within the next minute.**

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Poisson RV as a Limit

Suppose that in the binomial pmf b(x;n,p) we let n→∞ and p→0 in such a way that np approaches a value λ>0. Then
b(x;n,p)→p(x;λ)

As a rule of thumb, this approximation can safely be applied if n>50 and np<5.

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Example

Assume that the probability of any given page (of a certain publisher's book) containing at least one typographical error is 0.005 and errors are independent from page to page. Find the probability that one of this publisher's 400-page book will contain

- exactly one page with errors

- at most three pages with errors.