Chapter 5

Joint Probability Distributions

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Sum of Rvs

If X_i are iid with $E(X_i)=\mu$ and $V(X_i)=\sigma^2$, then

1. $E(X_1+X_2+\ldots+X_n)=n\mu$
2. $V(X_1+X_2+\ldots+X_n)=n\sigma^2$
3. The expectation of the sample mean is μ
4. The variance of the sample mean is σ^2/n

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Example

Roll a die 20,000 times.

- Find $E(T_o)$
- Find $V(T_o)$
- Find the expectation of the sample mean
- Find the variance of the sample mean

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Sum of Normal Rvs

If X_i are iid normal with $E(X_i)=\mu$ and $V(X_i)=\sigma^2$, then

- The sample mean is a normal rv with expectation μ and variance σ^2/n
- T_o is a normal rv with expectation $n\mu$ and variance $n\sigma^2$

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Example

Suppose the time that it takes a randomly selected rat to find its way through a maze is a normal rv with $\mu=1.5$ min and $\sigma=0.35$ min.

- Find the probability that the total time for five randomly selected rats is between 6 and 8 min.
- Find the probability that the average time for five randomly selected rats is at most 2 min.

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Central Limit Theorem

If X_i are iid with $E(X_i)=\mu$ and $V(X_i)=\sigma^2$, then

- The sample mean is an approximately normal rv with expectation μ and variance σ^2/n
- T_o is an approximately normal rv with expectation $n\mu$ and variance $n\sigma^2$

This is true for large values of n

The larger, the better the approximation.

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Example

Suppose the weight of packages (in gram) is not normally distributed with $\mu=100$ and $\sigma^2=25.6$. We randomly choose 1000 packages.

- **Find the probability that the total weight is between 99.8 and 100.3 kg.**
- **Find a number c such that the total weight is at least c with probability 0.99.**

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Example

Roll a die 20,000 times.

Find a number c such that the probability of the sum being between 70000-c and 70000+c is at least 0.99.

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