

MISSOURI S&T MISSOURI UNIVERSITY OF SCIENCE AND TECHNOLOGY

## Chapter 5

### Joint Probability Distributions

- 5.1 Jointly Distributed Random Variables
- 5.2 Expected Values, Covariance, and Correlation
- 5.3 Statistics and Their Distributions
- 5.4 The Distribution of the Sample Mean**
- 5.5 The Distribution of a Linear Combination

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## Sum of Rvs

If  $X_i$  are iid with  $E(X_i)=\mu$  and  $V(X_i)=\sigma^2$ , then

1.  $E(X_1+X_2+\dots+X_n)=n\mu$
2.  $V(X_1+X_2+\dots+X_n)=n\sigma^2$
3. The expectation of the sample mean is  $\mu$
4. The variance of the sample mean is  $\sigma^2/n$

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## Example

Roll a die 20,000 times.

- Find  $E(T_o)$
- Find  $V(T_o)$
- Find the expectation of the sample mean
- Find the variance of the sample mean

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## Sum of Normal Rvs

If  $X_i$  are iid **normal** with  $E(X_i)=\mu$  and  $V(X_i)=\sigma^2$ , then

- The sample mean is a **normal** rv with expectation  $\mu$  and variance  $\sigma^2/n$
- $T_o$  is a **normal** rv with expectation  $n\mu$  and variance  $n\sigma^2$

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## Example

Suppose the time that it takes a randomly selected rat to find its way through a maze is a normal rv with  $\mu=1.5$  min and  $\sigma=0.35$  min.

- Find the probability that the total time for five randomly selected rats is between 6 and 8 min.
- Find the probability that the average time for five randomly selected rats is at most 2 min.

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## Central Limit Theorem

If  $X_i$  are iid with  $E(X_i)=\mu$  and  $V(X_i)=\sigma^2$ , then

- The sample mean is an **approximately normal** rv with expectation  $\mu$  and variance  $\sigma^2/n$
- $T_o$  is an **approximately normal** rv with expectation  $n\mu$  and variance  $n\sigma^2$

This is true for large values of  $n$   
The larger, the better the approximation.

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### Example

Suppose the weight of packages (in gram) is not normally distributed with  $\mu=100$  and  $\sigma^2=25.6$ . We randomly choose 1000 packages.

- Find the probability that the total weight is between 99.8 and 100.3 kg.
- Find a number  $c$  such that the total weight is at least  $c$  with probability 0.99.

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### Example

Roll a die 20,000 times.

**Find a number  $c$  such that the probability of the sum being between  $70000-c$  and  $70000+c$  is at least 0.99.**

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