

MISSOURI S&T MISSOURI UNIVERSITY OF SCIENCE AND TECHNOLOGY

Chapter 5

Joint Probability Distributions

- 5.1 Jointly Distributed Random Variables
- 5.2 Expected Values, Covariance, and Correlation
- 5.3 Statistics and Their Distributions
- 5.4 The Distribution of the Sample Mean
- 5.5 The Distribution of a Linear Combination**

Stat 3115, Spring 2024 245

245

MISSOURI S&T MISSOURI UNIVERSITY OF SCIENCE AND TECHNOLOGY

Linear Combination of Rvs

If X_i are any rvs and a_i are any numbers, then

$$Y = \sum_{i=1}^n a_i X_i$$

is called a **linear combination** of the X_i

Stat 3115, Spring 2024 246

246

MISSOURI S&T MISSOURI UNIVERSITY OF SCIENCE AND TECHNOLOGY

Expectation

If X_i are any rvs and a_i are any numbers, then

$$E\left(\sum_{i=1}^n a_i X_i\right) = \sum_{i=1}^n a_i E(X_i)$$

is the expectation of the **linear combination**.

Stat 3115, Spring 2024 247

247

MISSOURI S&T MISSOURI UNIVERSITY OF SCIENCE AND TECHNOLOGY

Variance

If X_i are any rvs and a_i are any numbers, then

$$V\left(\sum_{i=1}^n a_i X_i\right) = \sum_{i=1}^n \sum_{j=1}^n a_i a_j \text{Cov}(X_i, X_j)$$

is the variance of the **linear combination**.
 For independent rvs X_i , this reduces to

$$V\left(\sum_{i=1}^n a_i X_i\right) = \sum_{i=1}^n a_i^2 V(X_i)$$

Stat 3115, Spring 2024 248

248

MISSOURI S&T MISSOURI UNIVERSITY OF SCIENCE AND TECHNOLOGY

Example

A gas station sells three grades of gasoline: regular, extra, and super, priced at \$2.61, \$2.69, and \$2.79 per gallon, respectively. Suppose the sales in gallons on a particular day are independent rvs with expectations 1000, 500, and 300 and standard deviations 100, 80, and 50, respectively.

Find the expectation and standard deviation of the revenue from sales on one day.

Stat 3115, Spring 2024 249

249

MISSOURI S&T MISSOURI UNIVERSITY OF SCIENCE AND TECHNOLOGY

Normal Rvs

If the X_i are independent and **normal** rvs, then any **linear combination** of the X_i is also a **normal** rv.

Stat 3115, Spring 2024 250

250

Example

A gas station sells three grades of gasoline: regular, extra, and super, priced at \$2.61, \$2.69, and \$2.79 per gallon, respectively. Suppose the sales in gallons on a particular day are independent rvs with expectations 1000, 500, and 300 and standard deviations 100, 80, and 50 respectively.

Assuming in addition that the three rvs are normally distributed, find the probability that revenue on a particular day exceeds \$4000.

