

MISSOURI S&T MISSOURI UNIVERSITY OF SCIENCE AND TECHNOLOGY

Chapter 7

Confidence Intervals

- 7.1 Basic Properties of Confidence Intervals
- 7.2 Large Sample Intervals for a Population Mean and Proportion
- 7.3 Intervals Based on a Normal Population
- 7.4 Intervals for the Variance of a Normal Population

Stat 3115, Spring 2024 252

252

MISSOURI S&T MISSOURI UNIVERSITY OF SCIENCE AND TECHNOLOGY

Example

Suppose X_1, X_2, \dots, X_n is a random sample from a normal population and σ is known.

Find an interval such that the probability that the true value of μ is inside this interval equals to 0.95.

$\bar{x} - 1.96\sigma/\sqrt{n}$ \bar{x} $\bar{x} + 1.96\sigma/\sqrt{n}$

Stat 3115, Spring 2024 253

253

MISSOURI S&T MISSOURI UNIVERSITY OF SCIENCE AND TECHNOLOGY

95% Confidence Interval

After observing x_1, x_2, \dots, x_n from a normal population with known σ , the interval

$$\left(\bar{x} - 1.96 \frac{\sigma}{\sqrt{n}}, \bar{x} + 1.96 \frac{\sigma}{\sqrt{n}} \right)$$

is called a 95% **confidence interval (CI)** for μ .

Stat 3115, Spring 2024 254

254

MISSOURI S&T MISSOURI UNIVERSITY OF SCIENCE AND TECHNOLOGY

Confidence Interval

After observing x_1, x_2, \dots, x_n from a normal population with known σ , the interval

$$\left(\bar{x} - z_{\alpha/2} \frac{\sigma}{\sqrt{n}}, \bar{x} + z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \right)$$

is called a $100(1-\alpha)\%$ **confidence interval (CI)** for μ .

Stat 3115, Spring 2024 255

255

MISSOURI S&T MISSOURI UNIVERSITY OF SCIENCE AND TECHNOLOGY

One-sided Confidence Intervals

After observing x_1, x_2, \dots, x_n from a normal population with known σ , the intervals

$$\left(-\infty, \bar{x} + z_{\alpha} \frac{\sigma}{\sqrt{n}} \right) \quad \left(\bar{x} - z_{\alpha} \frac{\sigma}{\sqrt{n}}, \infty \right)$$

are called two **one-sided $100(1-\alpha)\%$ confidence intervals** for μ .

Stat 3115, Spring 2024 256

256

MISSOURI S&T MISSOURI UNIVERSITY OF SCIENCE AND TECHNOLOGY

Example

A psychologist tests 51 people on their reaction times to a certain signal. We assume that this random variable is normal with known variance 0.04. Suppose the 51 measurements result in a sample mean of 0.8 seconds.

- Find a 95% confidence interval for μ .
- Find the two one-sided 95% confidence intervals for μ .
- Find a 99% confidence interval for μ .
- How large should n be so that the 95% confidence interval has length at most 0.02?

Stat 3115, Spring 2024 257

257