R-factor and Optimal Placement of Receivers for Enhancing Location Accuracy in an RSSI System

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***Abstract*— The fundamental cause of localization error in an indoor environment is fading and spreading of the radio signals due to scattering, diffraction, and reflection. This work proposes a new factor called R-factor to measure the localization error introduced by a receiver in a real-time location determination system (RTLS). Additionally, R-factor is shown to be reduced by using antenna or frequency diversity. Finally, R-factor is used to compute the optimal placement of receivers on the target area.**

I. INTRODUCTION

Location information of an asset is a key requirement in the network-centric environment. In an outdoor environment, Global Positioning Systems (GPS) have been very successful, however, lack of satellite coverage and unit cost have severely restricted the use of GPS for indoor positioning. Consequently, a wide variety of technologies such as Time of Arrival (ToA), Time Difference of Arrival (TDoA), Angle of Arrival (AoA), and Received Signal Strength Indicator (RSSI) of radio [1] and acoustic [2] waves have been proposed for indoor localization. Several factors including large positioning errors, cost of synchronization hardware, and time consuming calibration issues, have limited the widespread adoption of these technologies.

This work proposes a new parameter called the R-Factor to approximate the MSE of a receiver under varying levels of NLoS energy since no method in the literature is available that uses this metric to identify in real-time a subset of receivers under Ricean distribution for improving location accuracy. The major benefit of the proposed scheme is its applicability to both indoor and outdoor environments for localization by varying the Ricean K-factor. This paper begins by deriving the equation for MSE of the radial distance estimate obtained using a point estimator in a Ricean fading environment and shows that MSE degrades with R-factor and more importantly becomes unsatisfactory under NLoS conditions. Subsequently, R-factor is shown to be related to the localization error in the NLoS environment. Next, the use of diversity scheme combination using SC is shown to further reduce localization error. Thereafter, the placement of receivers is presented. Finally, the theoretical conclusions are verified using experimental results.

II. MEAN SQUARE ERROR OF RADIAL DISTANCE ESTIMATE

Theorem 1: (*MSE of radial distance estimate*) The MSE of radial distance estimate for an RSSI based RTLS under Ricean distribution is given be

$$MSE(R) = \frac{4\sigma_X^2}{n^2 l_0}\left(\frac{l_0}{A^2}\right)^{\frac{2}{n}+1}\left(1+\frac{(n+2)^2\sigma_X^2}{2n^2A^2}\right) \quad (1)$$

where σ_X^2 is the variance of the received signal, and A is the amplitude of the LoS component, n is the path loss distance coefficient and l_0 is a factor that takes into account the inefficiency of antenna, polarization effects etc. The above equation can be derived from a second order approximation of the Friis transmission equation based point estimator using Taylor series. The term $(n+2)^2/(2n^2A^2)$ in (1) is always less than one, hence MSE can be reduced substantially only by decreasing the term

$$\gamma = l_0^{\frac{2}{n}}\sigma_X^2 / A^{\frac{4}{n}+2} \quad (2)$$

where γ is called the R-factor (Receiver-Error Factor). For a localization receiver R-factor is related to the variance and the upper bound of the bias of (4) as

$$Var(R \mid A, \sigma_X) = \frac{4\gamma}{n^2} \propto \gamma; \quad B_R < \frac{(n+2)}{n^2}\sqrt{\frac{\gamma}{18}} \quad (3)$$

Remark 1 (*R-Factor and NLoS energy*): For a localization receiver, $d^n = l_0 / A^2$, R-factor can be written as $\gamma = (d^{n+2}/l_0)\sigma_X^2$. Therefore, for a known radial distance, R-factor is a measure of the energy in NLoS components.

III. DIVERSITY AND R-FACTOR

Diversity is a method to improve certain aspects of the received signal by using two or more communication channels. Two commonly used schemes for RTLS are spatial (multiple antennas single frequency) and frequency (single antenna multiple frequency) diversity. RTLS using two spatial diversity channels combined using selection combining (SC) was employed in [1] to mitigate signal fading. This is only possible if the selected diversity scheme ensures that the RSSI values from individual channels have minimal correlation among themselves, thereby minimizing the probability of simultaneous fading on all channels. Uncorrelated diversity scheme channels result in identical but independent (i.i.d) signal distribution at each channel. This section will show that SC diversity combination method reduces the R-factor and thus improves localization accuracy in an RTLS.

Definition 2: (*Selection Combining*): When a diversity channel with the highest signal amplitude value is selected as the X_{new}, the combination method is called Selection Combination. Hence $X_{new} = \max(X_1, X_2, \ldots, X_u)$.

Theorem 2: (*R-factor decrease with SC diversity***)** In an RSSI based RTLS, the R-factor at a localization receiver with u

diversity channels is greater than or equal to the R-factor obtained with *u+1* diversity channels when SC method is employed.

This was proved using numerical solution to find the R-factor and plot the variation of R-factor with diversity channels ,*u,* for this receiver. Figure 1 displays the R-factor for a receiver against *u* diversity channels combined using SC. The figure indicates that as the diversity channel count *u* increases, the R-factor drops rapidly, thus improving accuracy.

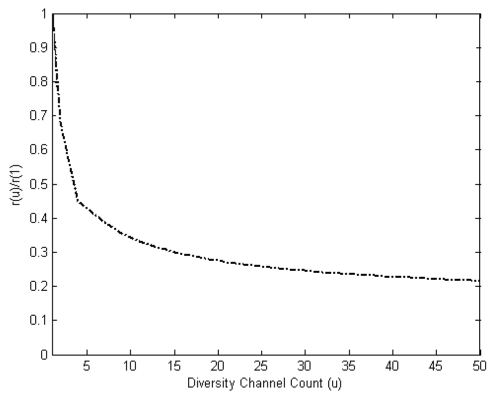

Figure 1. Radial distance estimation RMS error vs. actual radial distance

IV. R-Factor and Optimal Receiver Placement

Instability of localization coordinate estimate by node *i* is defined as the variance of the change in transmitter's estimates coordinates (β_x or β_y) with respect to a change in radial distance estimate (r_i) to that transmitter.

$$\xi_{(x,y)} = Var\left(\frac{d\beta_{(x,y)}}{dr_i}\right) \quad (4)$$

Theorem 3: (*Localization Instability and Span Angle*) For an RSSI based RTLS setup with three receivers, the location coordinate estimate obtained from an anchor node with the smallest span angle has the highest instability.

Hence least squares methodology for solving the transmitter location in an *n* receiver RTLS setup is highly unstable due to the presence of outliers. A robust localization method such as Least Median Square (LMS) is more appropriate in these situations. However, there is no closed form solution for LMS resulting in using numerical solutions that are $O(n^4)$ in complexity. To reduce the computation complexity from quadratic to a linear complexity, this paper proposes placing the receivers on the target location using a constrained Delaunay triangulation. Constrained Delaunay triangulation has the property that it can optimally divide a target area based on the minimum error constraints while providing a lower bound for the minimum span angle created by the geometry of an *n* receiver RTLS setup. Hence constrained Delaunay triangulation based placement sets an upper bound for transmitter location estimate instability. In the numerical solution for LMS when LMS candidates from the triangles formed by the constrained Delaunay triangulation are only included the computation complexity becomes *O(n)* for an n receiver RTLS.

V. Results and Analysis

A. Localization Experiments

First, the RTLS test-bed is addressed before introducing the PSS/TIX localization algorithm [2].

1) Testbed and Implementation

All experiments were conducted using G4-SSN motes developed at Missouri University of Science and Technology (Missouri S&T). G4-SSN motes use IEEE 802.15.4 wireless XBee transceivers from Maxstream. The Missouri S&T RTLS receiver with spatial diversity is shown in Figure 2 (b).

(a) (b)

Figure 2. (a) MST RTLS Transmitter (b) MST RTLS receiver

The receiver contains two independent wireless motes connected to quarter wave antennas. Each mote independently measured the RSSI on its antenna. To ensure identical but independent fading envelop PDF on the two antennas, they were spaced 25 cm (2λ) apart [1]. Each mote independently measured the RSSI on its antenna. The collected RSSI values were then sent wirelessly to a desktop machine acting as the RTLS coordinator. The coordinator computed the R-factor for the receivers and then selected three receivers with lowest R-factor, which were then passed to the PSS/TIX algorithm to obtain the location of the transmitter.

The transmitter shown in Figure 2 (a) is also a G4-SSN mote with a single quarter wave antenna. The test-bed shown in Figure 3 spans 13m by 12m and covers the entire floor of LAB 114 on the Engineering Research Laboratory (ERL) building at MST. A total of eight receivers marked R1 to R8 were placed on the target area as show in Figure 3. The positions of the receivers were selected to result in at least three localization receivers so that trilateration can be done.

Figure 3. Map and coordinate system of test bed

2) Location Determination Algorithm

The PSS/TIX algorithm developed by Gwon and Jain [2] was used to locate the position of the transmitter. This algorithm uses a heuristic method called Proximity in Signal Space (PSS) to generate an RSSI versus distance mapping curve. The RSSI values measured by a wireless receiver are then translated to radial distances based on this table lookup. The radial distances to the transmitter are measured by multiple receivers and then passed to a modified version of triangulation called Triangular Interpolation and eXtrapolation (TIX). The Gwon and Jain version of the TIX algorithm selects the three receivers with the highest RSSI and uses their radial distance to the transmitter to compute the *x-y* coordinates.

3) *Localization Results and Analysis*

To measure the advantage of using R-factor, three localization experiments were performed. In the first experiment, the PSS/TIX by Gwon and Jain [2] was replicated. In this experiment, the three receivers needed for TIX were selected by the coordinator based on highest RSSI values. In the second experiment, the coordinator computed R-factor for each receiver and the three receivers with lowest R-factor were selected. TIX algorithm was then applied to locate the transmitter. The final experiment combined the R-Factor and the TIX algorithm. The RSSI values of spatially diverse antennas were combined using SC, and the R-Factor was computed for the combined RSSI. Once again, the three receivers for the TIX algorithm were selected based on the lowest R-Factor values. The CDF plot of localization accuracy from eight sample locations around the target area for the three methods is shown in Figure 4.

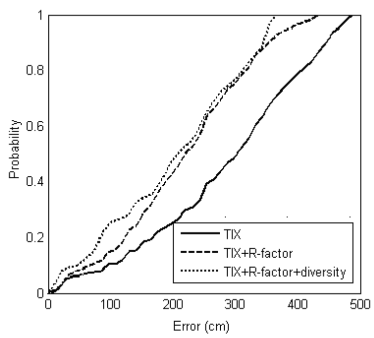

Figure 4. CDF of localization error

For each location, 50 localization measurements were collected, giving a total 400 localization error values to create the CDF plot. Table 1 presents the mean, the median, 90th percentile, and the standard deviation of the localization error.

TABLE I. SUMMARY OF LOCALIZATION ERROR LEVELS

Localization Method	**Localization Error (cm)**			
	Mean	***Median***	***90th percentile***	***Std. dev***
PSS/TIX	342	298	432	62.81
PSS/TIX with R-factor	267	214	335	40.32
PSS/TIX with R-factor and Spatial Diversity using SC	254	210	329	40.15

The mean error improved by 22%, the median error by 28%, and the 90th percentile by 22% from the PSS/TIX to the PSS/TIX with R-factor. Adding spatial diversity to the R-Factor improved the mean error by 26%, the median error by 30% and 90th percentile by 24% from the PSS/TIX. Although PSS/TIX scheme is employed as an illustration, other schemes can be deployed as well. Therefore proposed R-Factor improved the accuracy of the PSS/TIX localization scheme by selecting LoS receivers.

B. *Receiver Placement Simulation*

The optimal placement of receivers in a 16m x 16m rectangular room was generated using constrained Delaunay triangulation. Constrains were set to have a maximum of 1m localization error throughout the target area. Figure 4 shows the result of optimal placement algorithm.

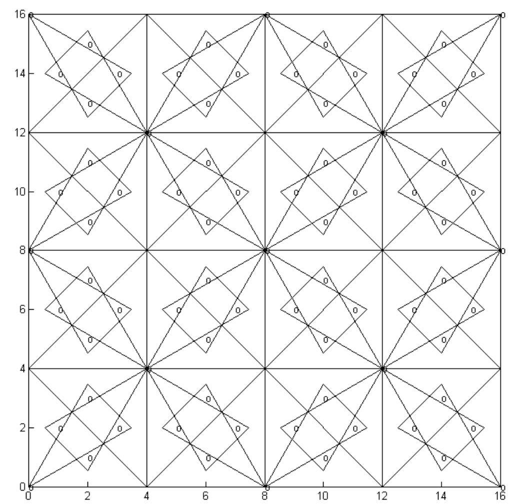

Figure 4. Optimal placement of receivers marked by '○' .

Total of 81 receivers were needed to cover the entire target area for 1m accuracy. With cheap RSSI based RTLS hardware, these requirement could be met in a cost effective way.

VI. CONCLUSIONS

This short paper presents a novel parameter called the R-Factor, and demonstrates that diversity channels combined using SC scheme was shown theoretically and experimentally to improve localization accuracy in an RSSI based RTLS. Experimental results show than an average 22% improvement in the mean localization accuracy when the R-factor was used in existing RTLS algorithms and 26% when diversity scheme with SC was applied. Though the proposed optimal placement algorithm currently requires 81 receivers to provide 1m accuracy, future work involves reducing the number of receivers while meeting the desired level of location accuracy.

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