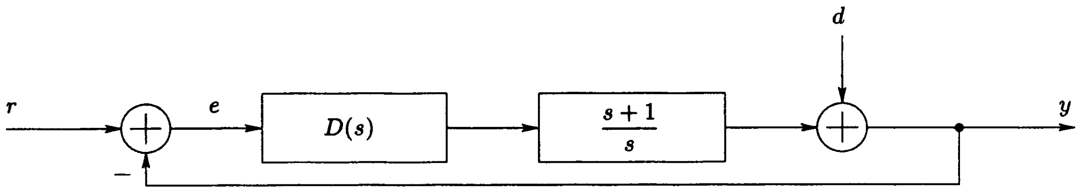
EE 231 **Exam#2** **Dec. 05, 2002**
75 minutes

1. (a) Describe and sketch the s-plane region specified by the following requirements for a second-order system described by $Y(s)/U(s) = \omega_n^2/(s^2 + 2\zeta\omega_n s + \omega_n^2)$. (15pts)

$$\begin{array}{rl} \text{Maximum percent overshoot} & 10\% \le M_p \le 20\%. \\ 2\% \text{ settling time} & t_{2\%s} \le 2\,\text{s}. \end{array}$$

   (b) Consider a second-order system with no zero, such that its poles are located in the region described as above. Determine the largest possible peak-time of the system. (15pts)

2. For the following feedback control system, design the simplest controller $D(s)$ that would track a step reference-input r and reject a sinusoidal disturbance-signal d with a frequency of 5 rad/s. (25pts)



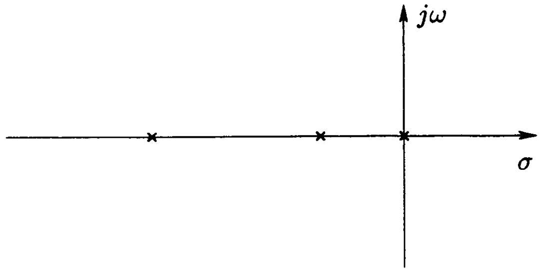
3. Consider a negative unity-feedback control system with the open-loop transfer function

$$G(s) = K\frac{1}{2s^4 + s^3 + 6s^2 + 3s}.$$

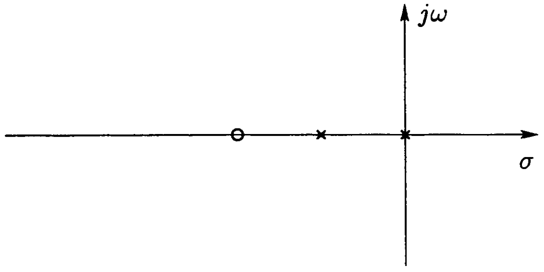
   (a) Determine the range of K for the asymptotical stability of the closed-loop system. (15pts)

   (b) Determine the steady-state errors for the unit-ramp and the unit-parabolic inputs. (10pts)

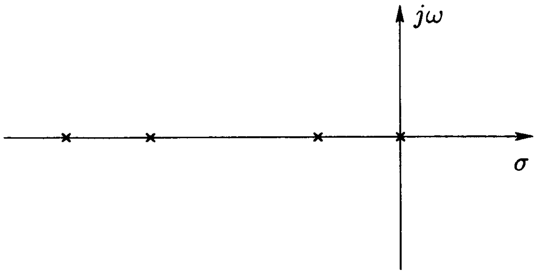
4. For the following open-loop pole/zero locations, sketch *expected* root-locus diagrams. *Do not* determine any features of the diagram, except the asymptote angles. Simply show the expected shapes of all the root-locus branches including the angles of departure from the real axis. (20pts)



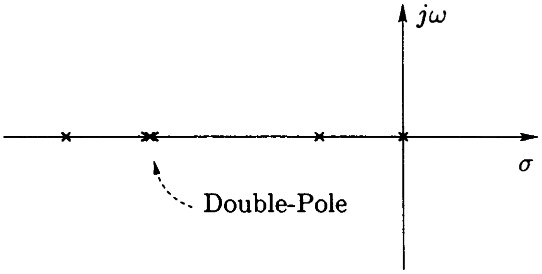
(a)



(b)



(c)



(d)

EE 231

Exam#2
Solutions

Dec. 05, 2002

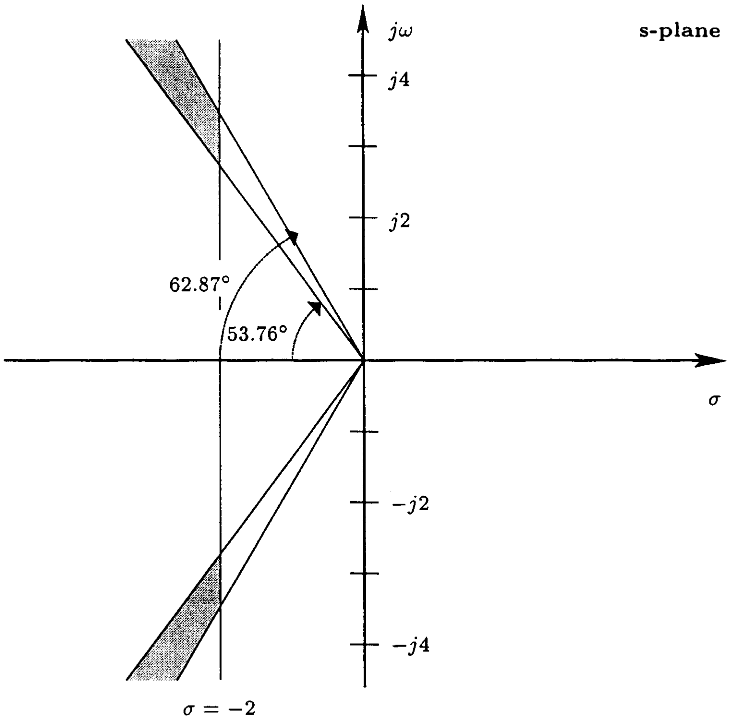
1. (a) Describe and sketch the s-plane region specified by the following requirements for a second-order system described by $Y(s)/U(s) = \omega_n^2/(s^2 + 2\zeta\omega_n s + \omega_n^2)$.

Maximum percent overshoot $10\% \leq M_p \leq 20\%$.
2% settling time $t_{2\%s} \leq 2\,\text{s}$.

Solution:

Given Specifications	System Constraints	Geometrical Representations
$10\% \leq M_p \leq 20\%$.	$0.1 \leq e^{-(\zeta/\sqrt{1-\zeta^2})\pi} \leq 0.2$, $\frac{\lvert \ln(0.2) \rvert}{\sqrt{(\ln(0.2))^2 + (\pi)^2}} \leq \zeta \leq \frac{\lvert \ln(0.1) \rvert}{\sqrt{(\ln(0.1))^2 + (\pi)^2}}$, or $0.46 \leq \zeta \leq 0.59$; since $M_p = e^{-(\zeta/\sqrt{1-\zeta^2})\pi}$, and $\zeta = \lvert \ln(M_p) \rvert / \sqrt{(\ln(M_p))^2 + (\pi)^2}$.	$\cos^{-1}(0.59) \leq \alpha \leq \cos^{-1}(0.46)$ or $53.76^\circ \leq \alpha \leq 62.87^\circ$, where $\alpha = \cos^{-1}(\zeta)$ is the angle measured from the negative real axis.
$t_{2\%s} \leq 2\,\text{s}$.	$\frac{4}{\sigma_o} \leq 2$, or $\sigma_o \geq 2$; since $t_{2\%s} = 4/\sigma_o$.	$\sigma \leq -2$, since the poles are at $s = -\sigma_o \pm j\omega_d$

The shaded region describes the region specified by the given requirements.



(b) Consider a second-order system with no zero, such that its poles are located in the region described as above. Determine the largest possible peak-time of the system.

Solution: The peak-time of the system is given by

$$t_p = \frac{\pi}{\omega_d}.$$

The largest peak-time is when we have the smallest ω_d. From the shaded region of the sketch in the previous part, we realize that the smallest possible imaginary value of the poles is at the intersection of the radial line with the angle of 53.76° with respect to the negative real axis and the vertical line at $\sigma = -2$. From the geometry, we determine that

$$\tan(53.76^\circ) = \frac{\omega_{d_{\min}}}{2},$$

and

$$t_{p_{\max}} = \frac{\pi}{\omega_{d_{\min}}} = \frac{\pi}{2\tan(53.76^\circ)};$$

or the largest possible peak time of the system is 1.15 s.

#2

To track a stepinput $D(s)\frac{s+1}{s} = \frac{1}{s}(\cdots)$

already exists

Reject sinusoidal disturbance $D(s)\frac{s+1}{s} = \frac{1}{s^2+5^2}(\cdots)$

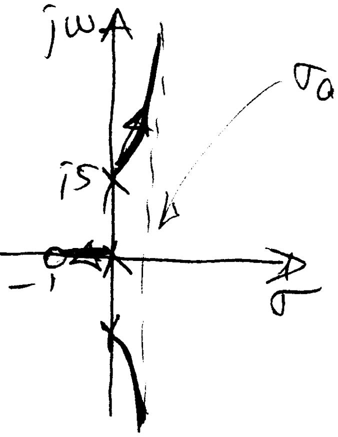
we need to include this term

So $D(s) = \frac{1}{s^2+25}D'(s)$ for some $D'(s)$

⊛ The simplest $D'(s)=K$, but

Open-loop gain $= \frac{K(s+1)}{s(s^2+25)}$

Root locus Diagram



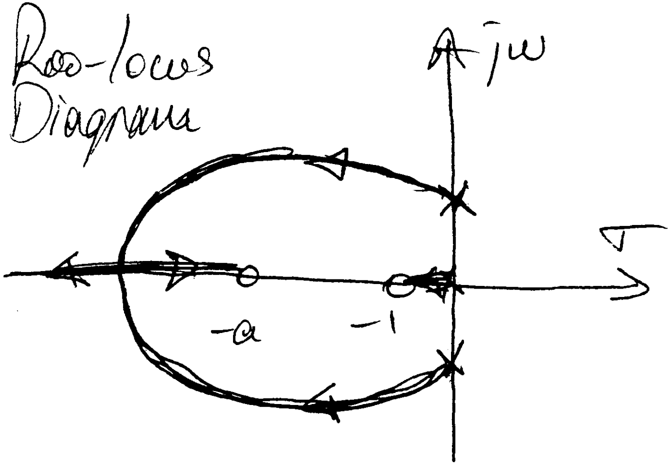
$\sigma_a = \frac{0+j5-j5-(-1)}{3-1} = \frac{1}{2}$

unstable for all K

(*) Since we have 2 poles in D(s) already, we may include a zero in D(s) or let $D(s) = K\frac{s+a}{s^2+25}$, then

$$\text{Open-loop gain} = \frac{K(s+1)(s+a)}{s(s^2+25)}$$

Root-locus Diagram



When a > 1 and K>0 we realize that the system is stable

Therefore One set of possible controllers is $D(s) = K\frac{s+a}{s^2+25}$ where

a > 1 and K > 0.

3. Consider a negative unity-feedback control system with the open-loop transfer function

$$G(s) = K\frac{1}{2s^4 + s^3 + 6s^2 + 3s}.$$

(a) Determine the range of K for the asymptotical stability of the closed-loop system.

Solution: The stability of the closed-loop system can be determined using the Routh-Hurwitz's stability criterion on the characteristic polynomial. From the characteristic equation, $1+G(s) = 0$,

$$1 + K\frac{1}{2s^4 + s^3 + 6s^2 + 3s} = 0,$$

or

$$2s^4 + s^3 + 6s^2 + 3s + K = 0.$$

The Routh-Hurwitz table for the system becomes as given below.

s^4	2	6	K
s^3	1	3	
s^2	$-\frac{(2)(3) - (1)(6)}{1} = 0$	K	
s			
1			

We have encountered the case where the first element of one of the Routh-Hurwitz table rows is zero and there are other nonzero elements on the same row. In this case, there are two possible methods to finish up the Routh-Hurwitz table.

First Method:

One method is to replace the leading zero with $\varepsilon > 0$, and let $\varepsilon \searrow 0$ for determining the stability.

s^4	2	6	K
s^3	1	3	
s^2	$-\frac{(2)(3) - (1)(6)}{1} = \cancel{0} \nearrow \varepsilon$	K	
s	$-\frac{(1)(K) - (\varepsilon)(3)}{\varepsilon}$		
1	K		

The Routh-Hurwitz's stability criterion implies the following conditions as $\varepsilon \searrow 0$.

i. $\lim_{\varepsilon \searrow 0} (-(K - 3\varepsilon)/\varepsilon) > 0$, or $K < 0$.

ii. $K > 0$.

Since the two conditions do not have a common region or a non-empty intersection, there is no value of K that will stabilize the system asymptotically.

Second Method:

Another method is to apply a bijection mapping on the roots of the characteristic polynomial such that they do not map from the left-half plane to the right-half plane or vice versa. One such mapping is $s \mapsto 1/s'$ for $s \neq 0$. So the roots of the characteristic equation

$$2s^4 + s^3 + 6s^2 + 3s + K = 0,$$

and the roots of

$$2(1/s')^4 + (1/s')^3 + 6(1/s')^2 + 3(1/s') + K = 0$$

or

$$2 + s' + 6s'^2 + 3s'^3 + Ks'^4 = Ks'^4 + 3s'^3 + 6s'^2 + s' + 2 = 0$$

have the same stability characteristics. So, we can apply the Routh-Hurwitz criterion on the mapped polynomial.

$$\begin{array}{c|ccc}
s'^4 & K & 6 & 2 \\
s'^3 & 3 & 1 & \\
s'^2 & -((K)(1) - (3)(6)) = 18 - K & 6 & \\
s' & -\dfrac{(3)(6) - (18 - K)(1)}{18 - K} = \dfrac{-K}{18 - K} & & \\
1 & 6 & &
\end{array}$$

The Routh-Hurwitz's stability criterion on the new polynomial implies the following conditions.

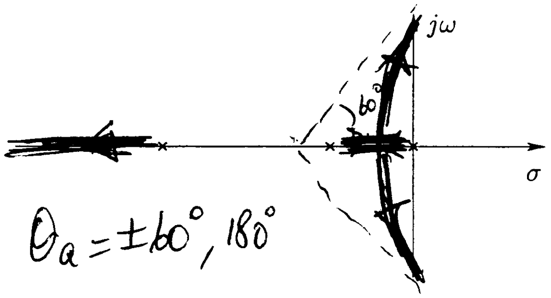
i. $K > 0$.
ii. $18 - K > 0$ or $K < 18$.
iii. $\big(-K/(18 - K)\big) > 0$, but since $18 - K > 0$ from the previous condition $-K > 0$, or $K < 0$.

Similar to the first method, the conditions do not have a a non-empty intersection, so there is no value of K that will stabilize the system asymptotically.

(b) Determine the steady-state errors for the unit-ramp and the unit-parabolic inputs.

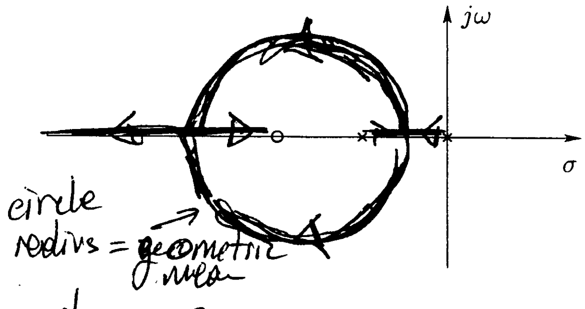
Solution: Since the system is unstable for any value of K, the output will increase exponentially. As a result, the error between an exponentially increasing signal and a step or ramp signal will be infinity. The expressions for steady-state errors cannot be used in this case, because those expressions are valid for stable systems. Therefore, the steady-state errors for the unit-ramp and the unit-parabolic inputs are infinity.

4. For the following open-loop pole/zero locations, sketch *expected* root-locus diagrams. *Do not* determine any features of the diagram, except the asymptote angles. Simply show the expected shapes of all the root-locus branches including the angles of departure from the real axis.



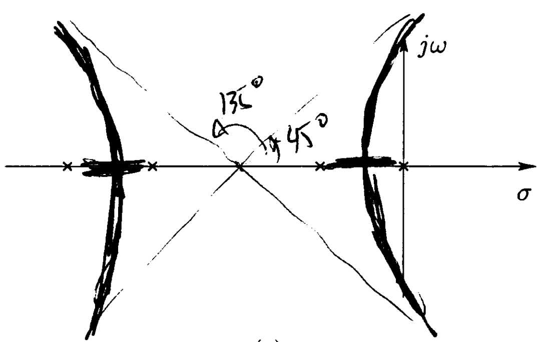
$\theta_a = \pm 60°, 180°$

(a)



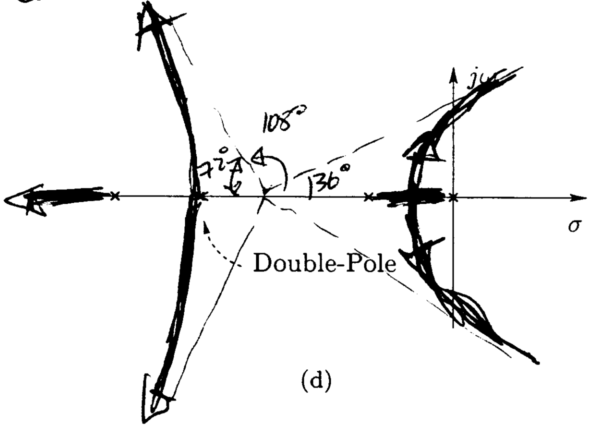
circle
radius = geometric mean
center = zero

(b)



(c)

$\theta_a = \pm 45°, \pm 135°$



(d)

$\theta_a = \pm 36°, \pm 108°, 180°$