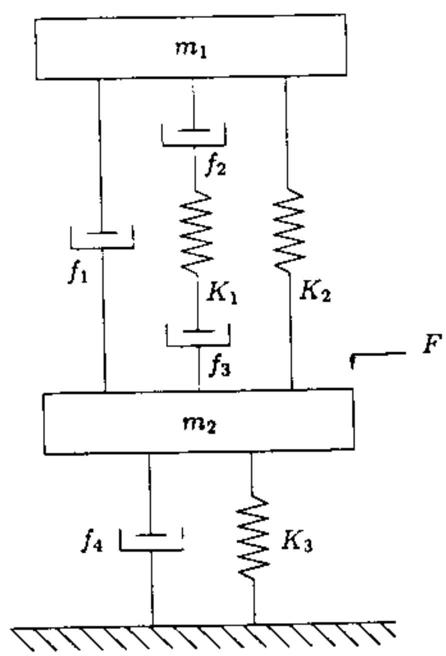
EE 231

Exam#1
50 minutes

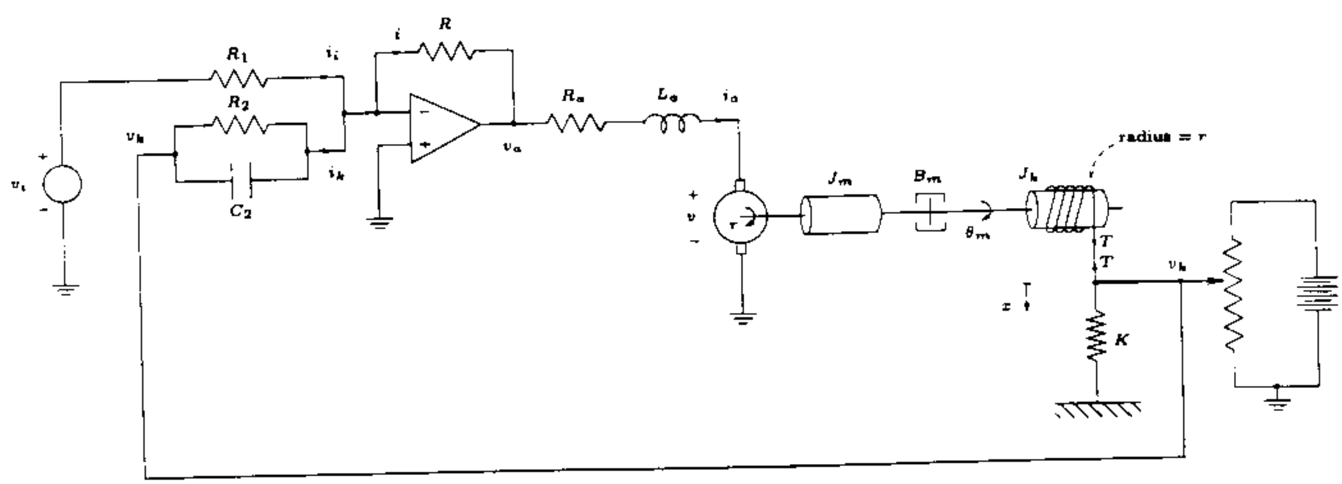
Oct. 27, 1993

1. For the mechanical system shown below,



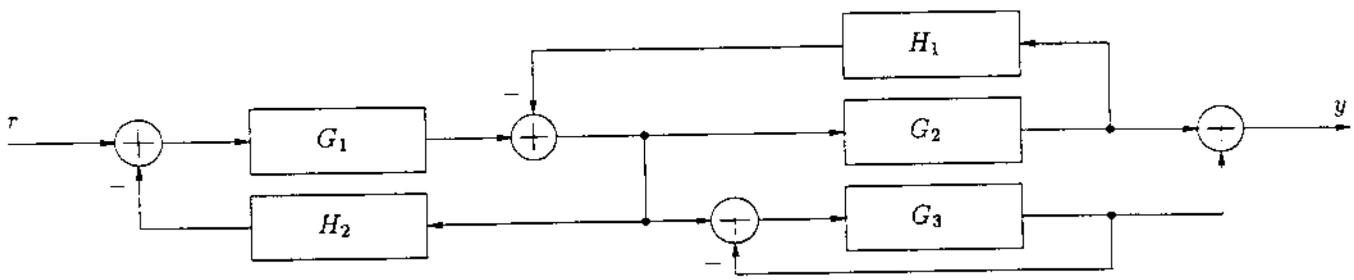
obtain *either* the force-voltage *or* the force-current analog of the system. (25pts)

2. The tension on an elastic cord, which is represented by a spring with a spring constant K, is adjusted by a motor as shown below.



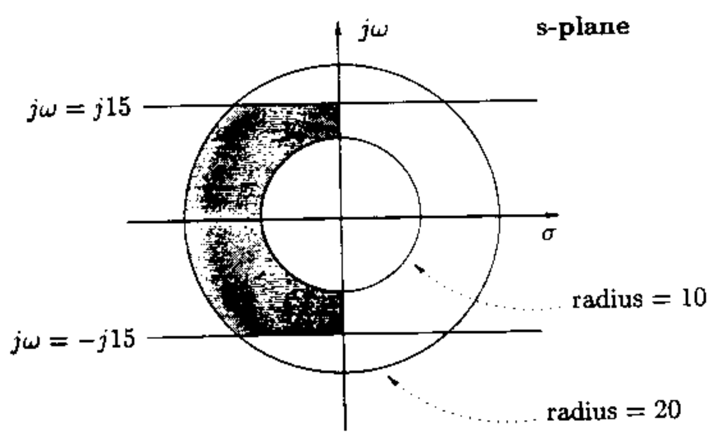
The tension on the cord is detected by the displacement x, and a variable resistor is used to obtain a voltage v_k proportional to the displacement, such that $v_k = -k_k x$. Draw the most detailed block diagram of the system, where v_i is the input, and T is the output. Show all the variables v_i, i_i, v_k, i_k, i, v_a, i_a, τ, v, θ_m, T, and x on the block diagram. (30pts)

3. For the block diagram given below,

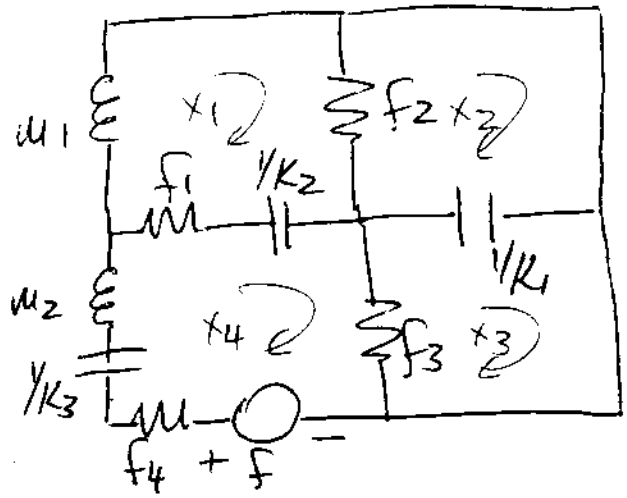


determine the transfer function *either* by block diagram reduction, *or* by Mason's formula. Show your work clearly. (25pts)

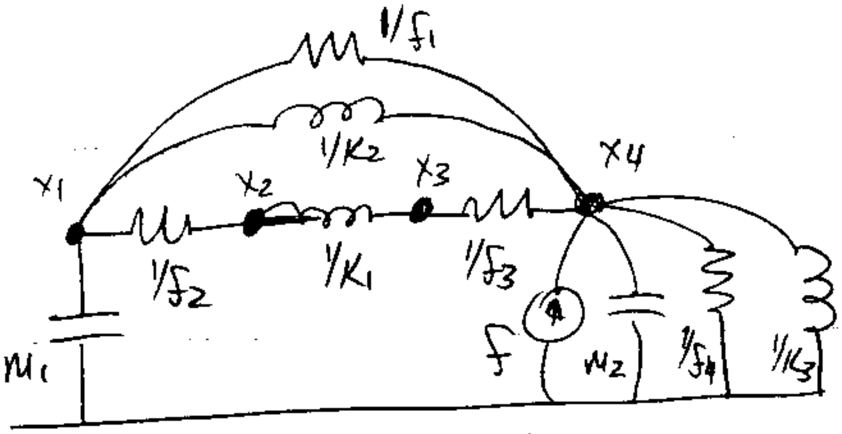
4. Obtain the necessary inequalities to describe the poles in the shaded region below in terms of only ζ and ω_n of a second-order system described by $Y(s)/U(s) = \omega_n^2/(s^2 + 2\zeta\omega_n s + \omega_n^2)$. (20pts)



#1

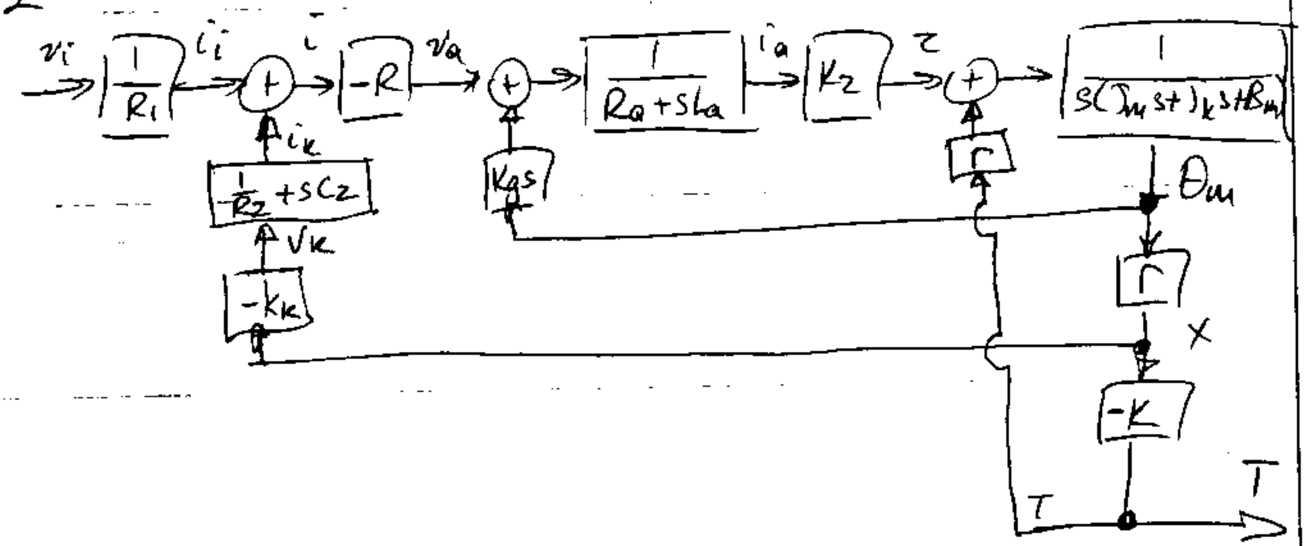


m_1
x_1
f_2
x_2
f_1
$1/K_2$
$1/K_1$
m_2
x_4
f_3
x_3
$1/K_3$
f_4
+ f −



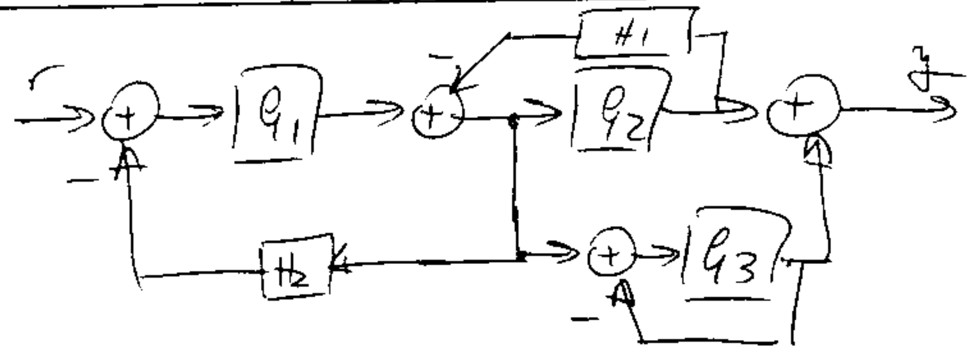
$1/f_1$
$1/K_2$
x_1
x_2
x_3
x_4
$1/f_2$
$1/K_1$
$1/f_3$
m_1
f
m_2
$1/f_4$
$1/K_3$

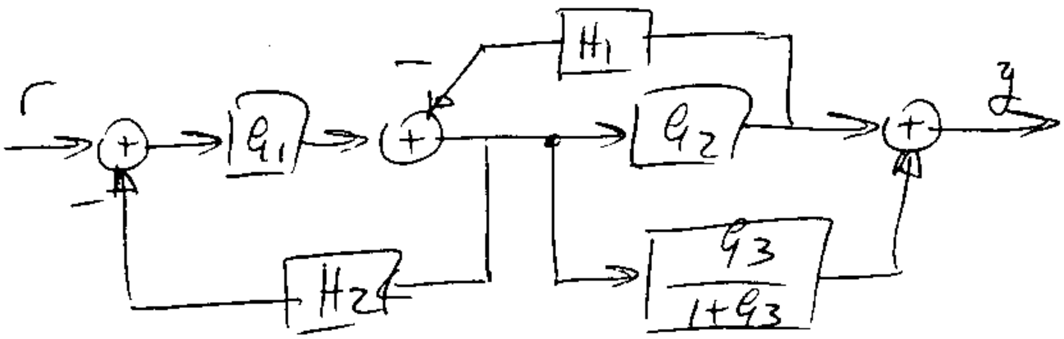
#2

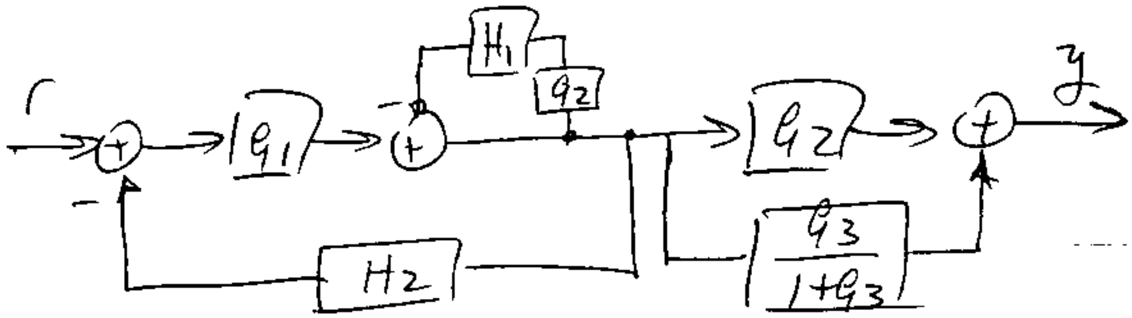


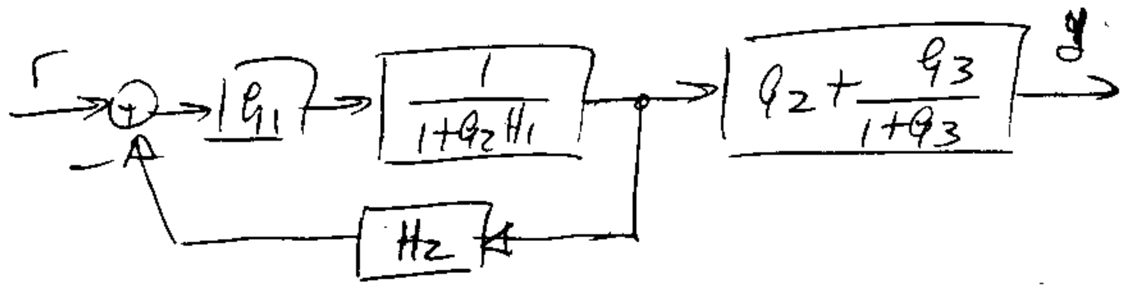
v_i
$\frac{1}{R_1}$
i_i
i
−R
v_a
$\frac{1}{R_a+sL_a}$
i_a
K_2
τ
$\frac{1}{s(J_m s+)_k s+B_m}$
i_K
$\frac{1}{R_2}+sC_2$
v_K
$-K_K$
$K_g s$
Γ
θ_m
Γ
x
−K
T
T

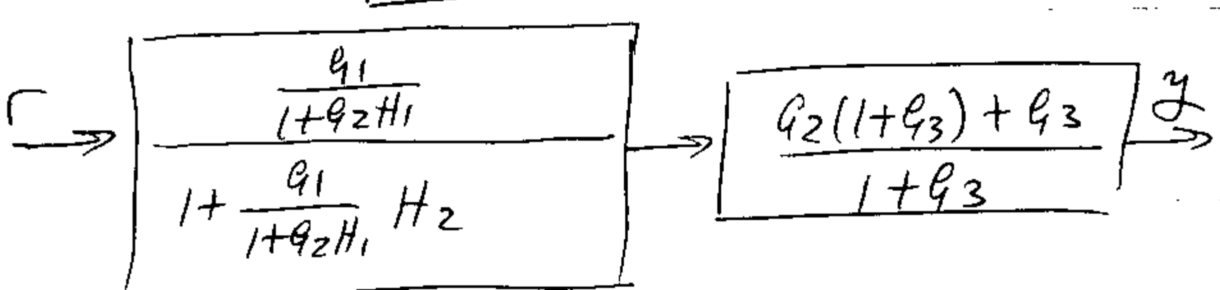
#3





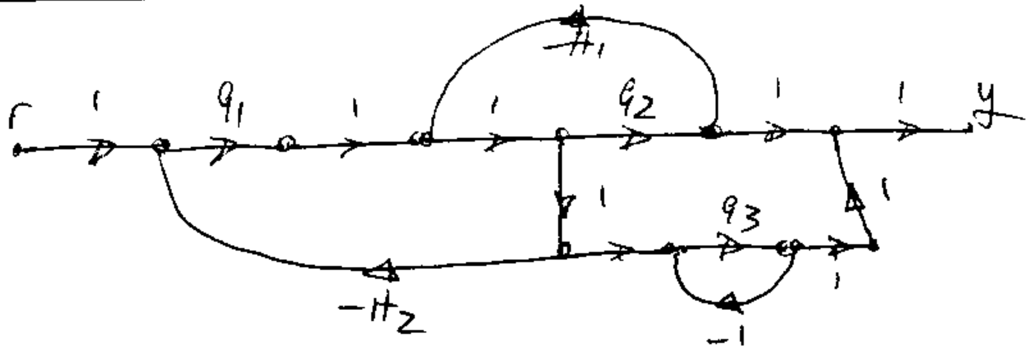






$$\frac{Y(s)}{R(s)} = \frac{G_1}{1+G_2H_1+G_1H_2} \cdot \frac{G_2(1+G_3)+G_3}{1+G_3}$$

$$= \frac{G_1G_2(1+G_3)+G_1G_3}{(1+G_3)(1+G_2H_1+G_1H_2)}$$



$F_1 = G_1 G_2$

$F_2 = G_1 G_3$

$L_1 = -G_2 H_1$

$L_2 = -G_1 H_2$

$L_3 = -G_3$

NON-TOUCHING LOOPS : $L_1 \& L_3$, $L_2 \& L_3$

L_1 is on $F_1 \& F_2$

L_2 is on $F_1 \& F_2$

L_3 is on F_2

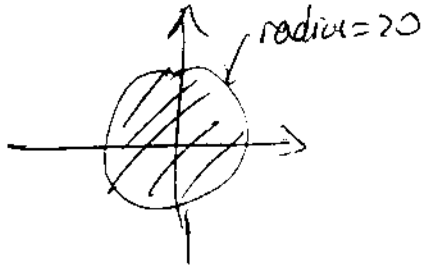
$$\Delta = 1 - (L_1 + L_2 + L_3) + (L_1 L_3 + L_2 L_3)$$
$$= 1 + G_2 H_1 + G_1 H_2 + G_3 + G_2 G_3 H_1 + G_1 G_3 H_2$$

$$\Delta_1 = \Delta \Big|_{L_1 = L_2 = 0} = 1 - L_3 = 1 + G_3$$

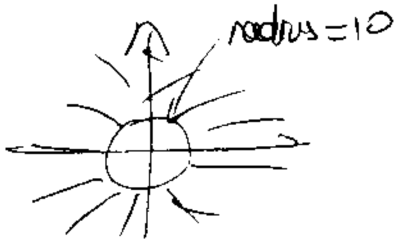
$$\Delta_2 = \Delta \Big|_{L_1 = L_2 = L_3 = 0} = 1$$

$$\frac{Y(s)}{R(s)} = \frac{1}{\Delta}(F_1\Delta_1 + F_2\Delta_2) = \frac{G_1 G_2 (1 + G_3) + G_1 G_3}{1 + G_2 H_1 + G_1 H_2 + G_3 + G_2 G_3 H_1 + G_1 G_3 H_2}$$

#4

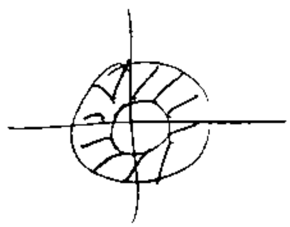


$\Longrightarrow \quad \omega_n \le 20$

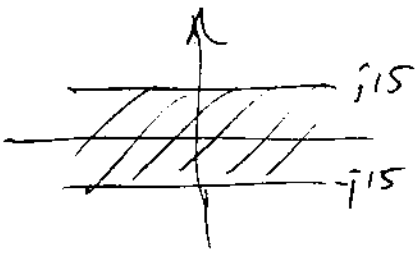


$\Longrightarrow \quad \omega_n \ge 10$

so



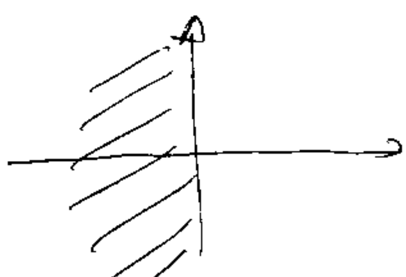
$\Longrightarrow \quad 10 \le \omega_n \le 20$



$\Longrightarrow$ when $-1 \le \zeta \le 1$ (i.e. complex)

$\omega_d \le 15$

$\omega_n \sqrt{1-\zeta^2} \le 15$



$\Longrightarrow \quad \zeta \ge 0$

Intersection of all these regions give the desired region

so (a) when $0 \le \zeta \le 1$ (i.e. complex poles)

$10 \le \omega_n \le 20$ and $\omega_n \sqrt{1-\zeta^2} \le 15$

(b) when $\zeta \ge 1$ (i.e. real poles)

$$\left.\begin{aligned} -\zeta\omega_n - \sqrt{\zeta^2-1}\,\omega_n &\ge -20 \\ \text{and} \quad -\zeta\omega_n + \sqrt{\zeta^2-1}\,\omega_n &\le -10 \end{aligned}\right\} \Rightarrow \begin{aligned} (\zeta+\sqrt{\zeta^2-1})\,\omega_n &\le 20 \\ (\zeta-\sqrt{\zeta^2-1})\,\omega_n &\ge 10 \end{aligned}$$