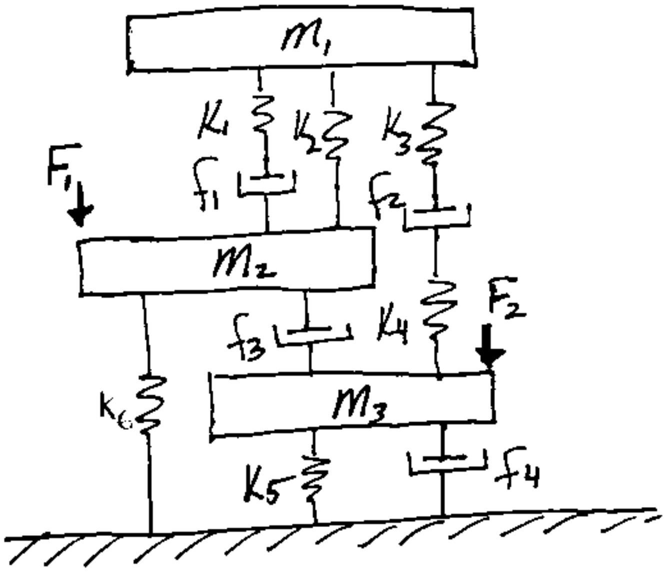
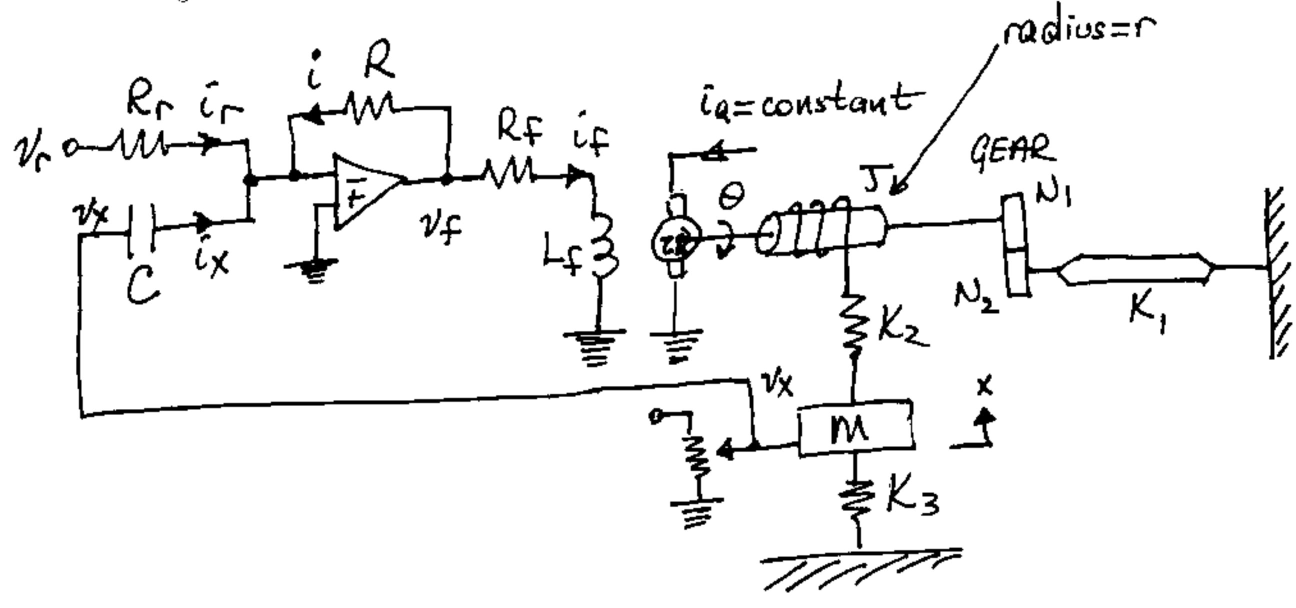
EE 231 | Exam#1 | Mar. 13, 1996

50 minutes

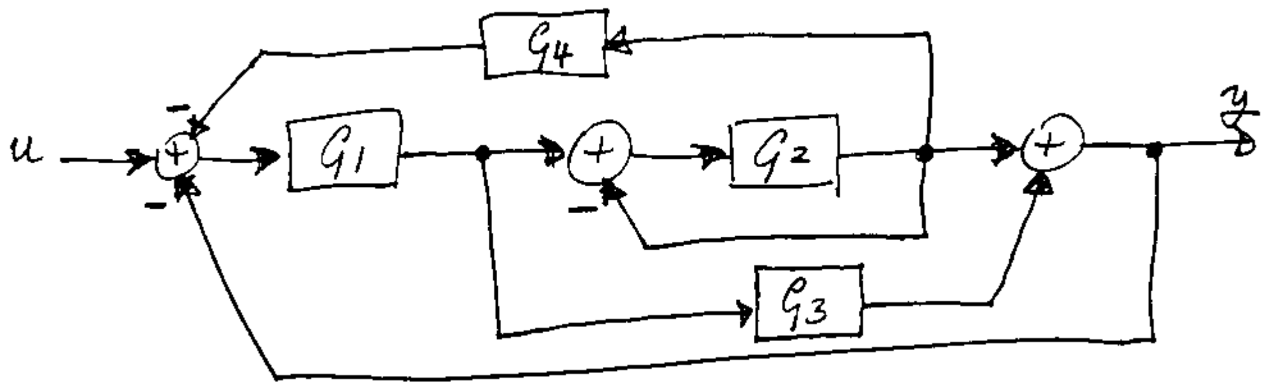
1. For the mechanical system shown below, obtain *either* the force-voltage *or* the force-current analog of the system. (25pts)



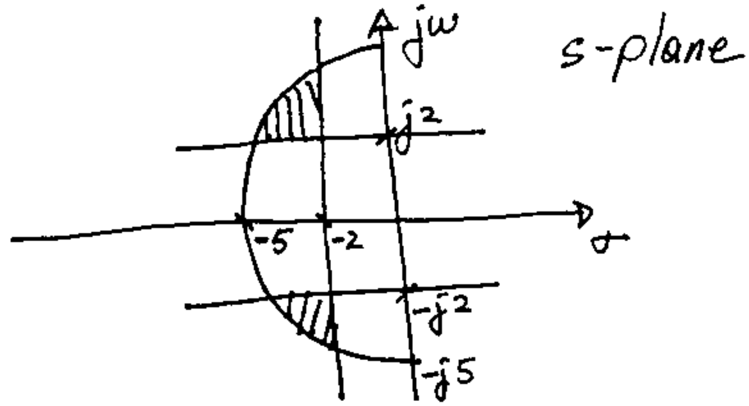
2. In the following system, the location of a mass is controlled by a motor control system. Assuming that the input and the output are v_r and x, respectively, and the voltage from the variable resistor is linearly related to the location of the mass, such that $v_x = k_x x$; obtain a detailed block diagram of the system without reducing or combining the equations, and show the variables v_r, i_r, v_x, i_x, i, v_f, i_f, τ, θ, and x on the block diagram. (30pts)



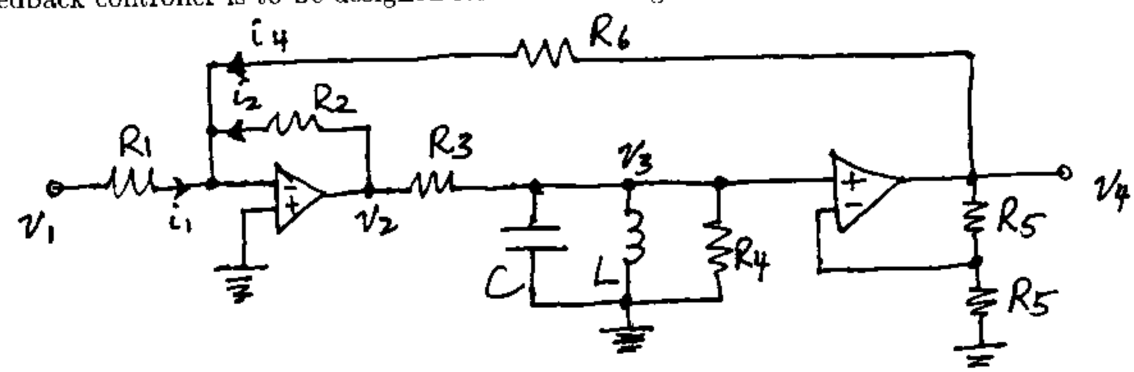
3. For the block diagram given below, determine the transfer function *either* by block diagram reduction, *or* by Mason's formula. Show your work clearly. (25pts)



4. Obtain the necessary inequalities to describe the poles in the shaded region below in terms of only ζ and ω_n of a second-order system described by $Y(s)/U(s) = \omega_n^2/(s^2 + 2\zeta\omega_n s + \omega_n^2)$. (20pts)



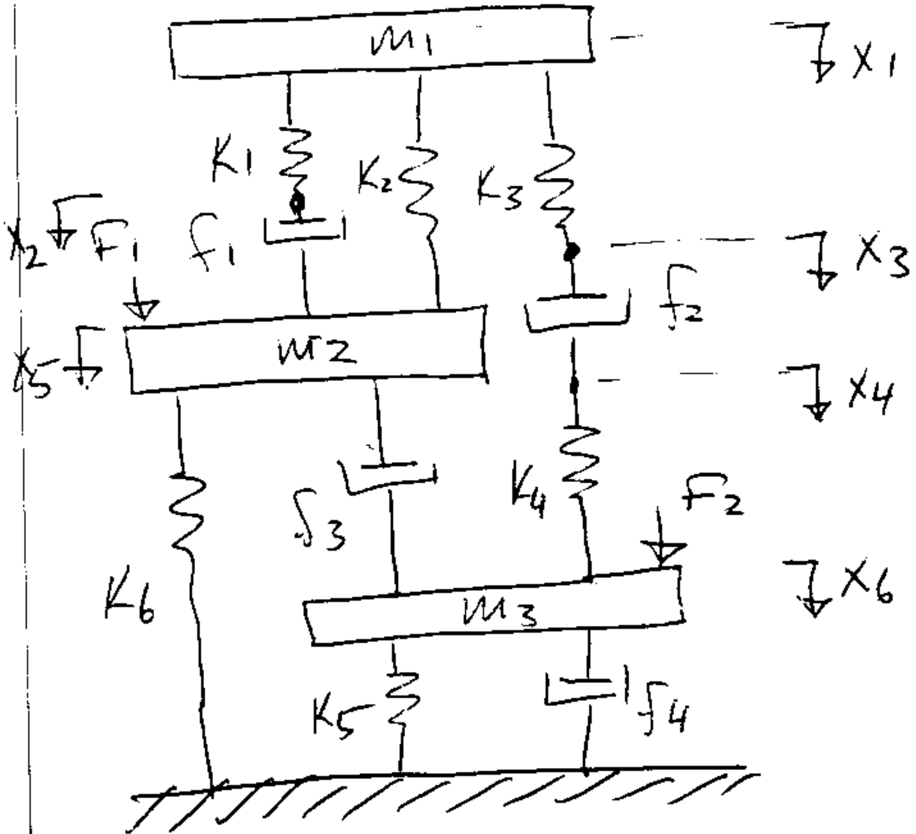
5. A feedback controller is to be designed for the following circuit.



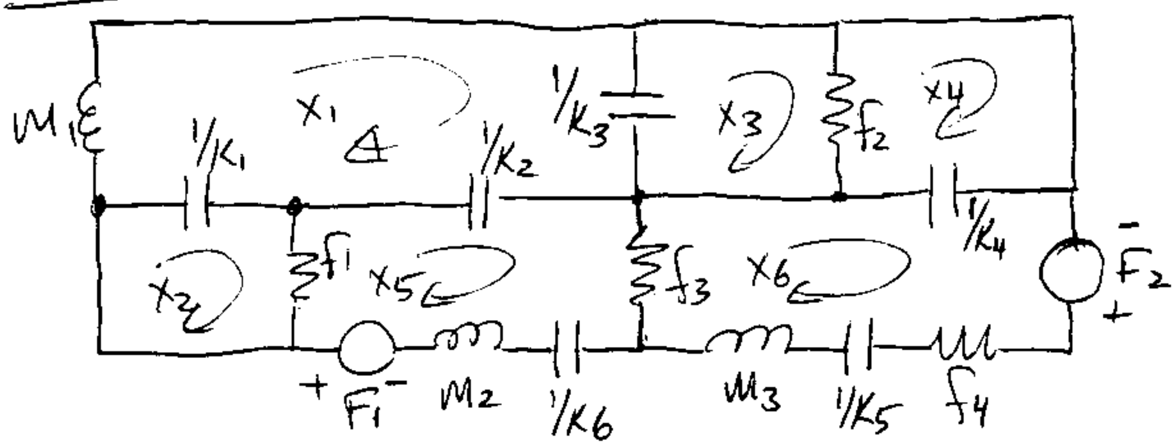
(a) First, determine its block diagram, such that the variables v_1, i_1, i_4, i_2, v_2, v_3, and v_4 are clearly shown. Then, obtain its transfer function from the block diagram, where v_1 is the input, and v_4 is the output. (+25pts)

(b) Assuming that $R_1 = 5\Omega$, $R_3 = 2\Omega$, $R_4 = 4\Omega$, $R_5 = 1\Omega$, $R_6 = 5\Omega$, $L = 4\,\text{H}$, and $C = 1\,\text{F}$; design for R_2, such that the $t_{5\%s} \approx 40/3\,\text{s}$. (+15pts)

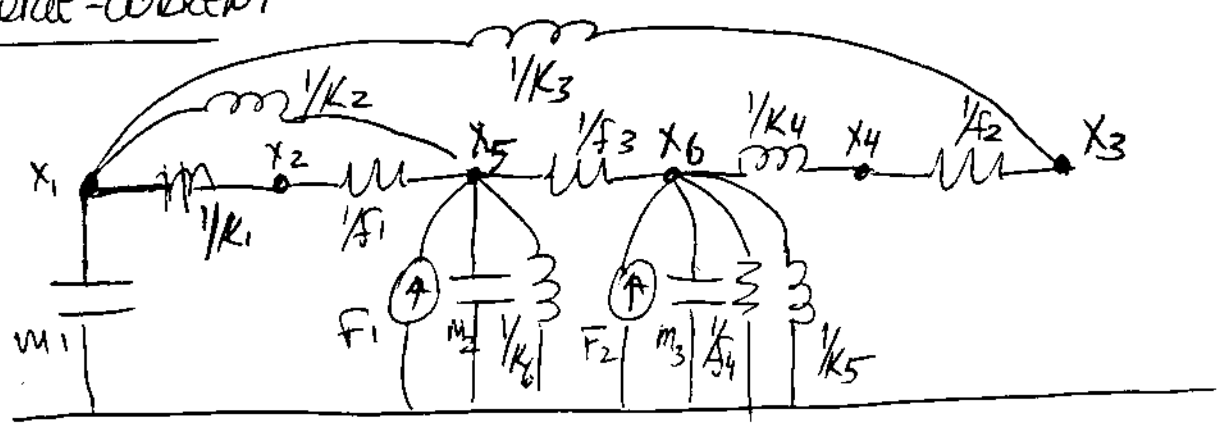
#1



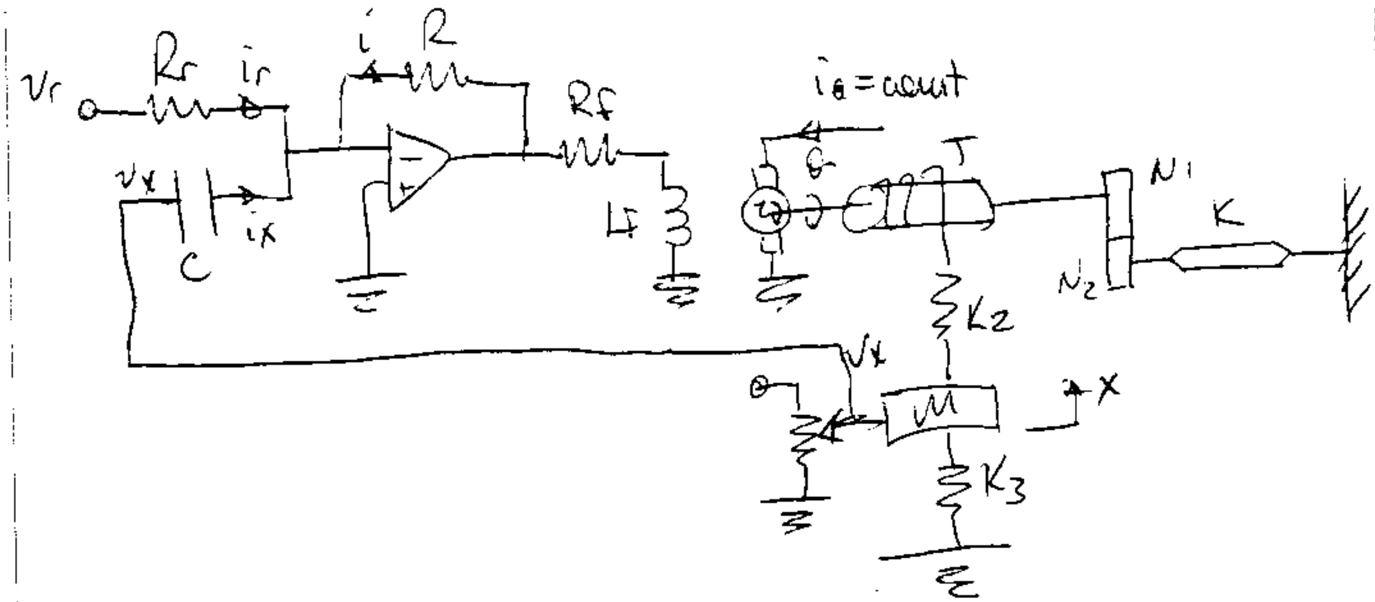
FORCE-VOLTAGE



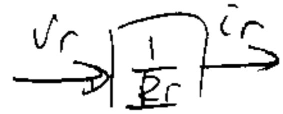
FORCE-CURRENT

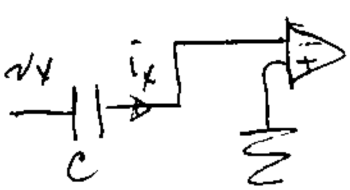


#2

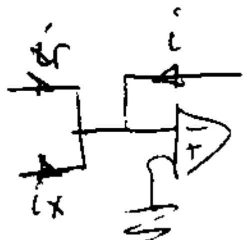


$$i_r = \frac{v_r}{R_r}$$



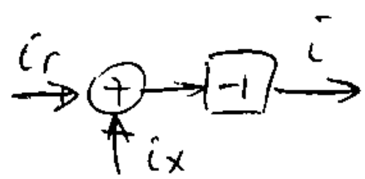


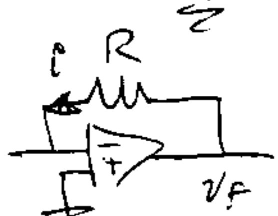
$$i_x = \frac{v_x}{1/sC} = sC\, v_x$$



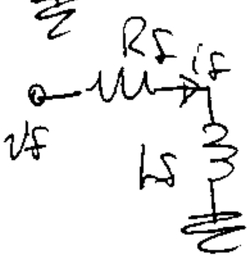
$$i + i_r + i_x = 0$$

$$i = -(i_r + i_x)$$

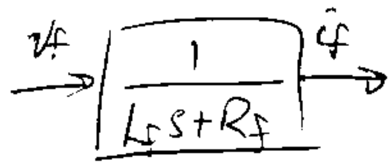


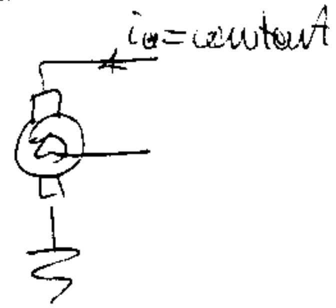


$$v_f = R i$$

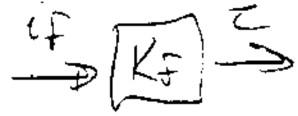


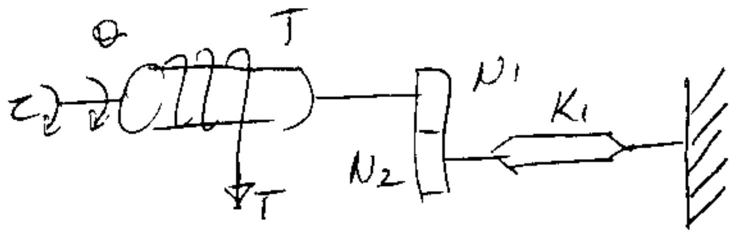
$$i_f = \frac{1}{L_f s + R_f} v_f$$





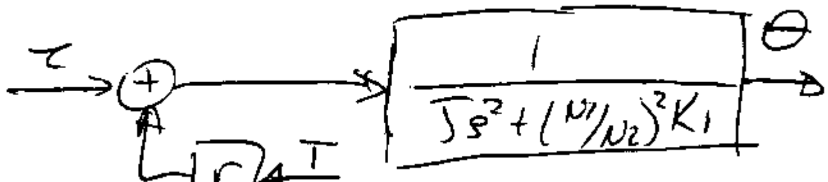
$$\tau = K_f i_f$$

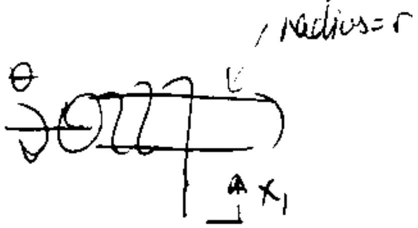




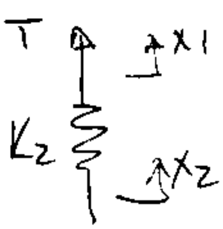
$$Js^2\theta = \tau + rT - \left(\frac{N_1}{N_2}\right)^2 K_1 \theta$$

$$\theta = \frac{1}{Js^2 + (N_1/N_2)^2 K_1}(\tau + rT)$$



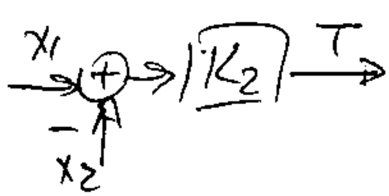


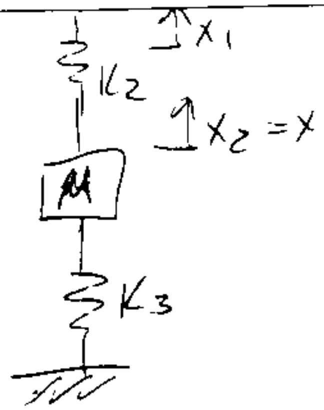
$$x_1 = -r\theta$$



$$0 = T - K_2(x_1 - x_2)$$

$$T = K_2(x_1 - x_2)$$

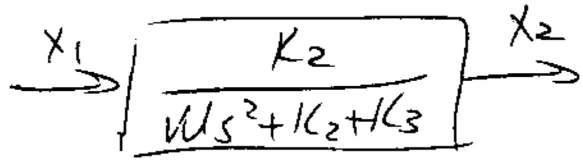


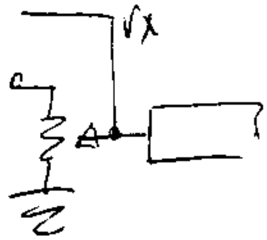


$$Ms^2X_2 = -K_2(X_2 - X_1) - K_3X_2$$

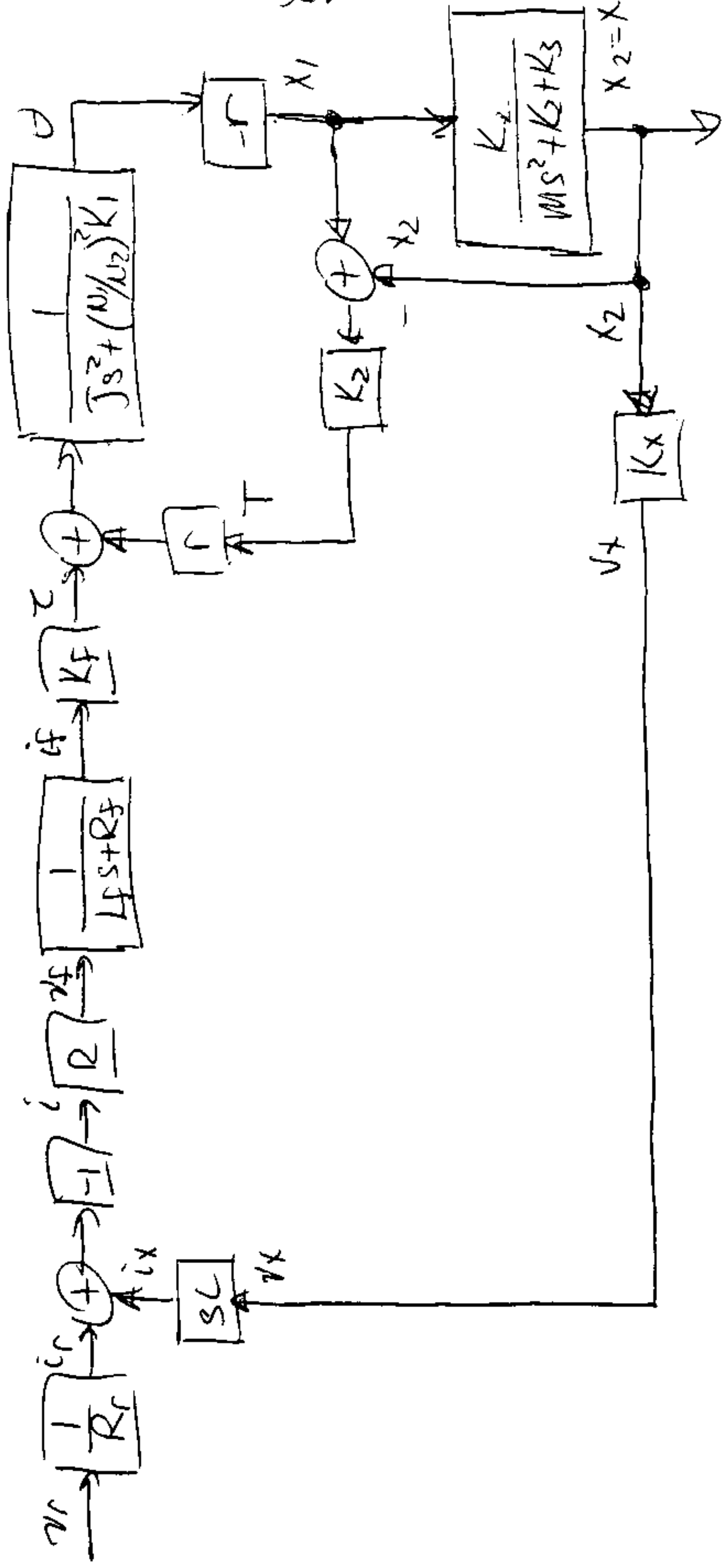
$$(Ms^2 + K_2 + K_3)X_2 = K_2X_1$$

$$X_2 = \frac{K_2}{Ms^2 + K_2 + K_3} X_1$$

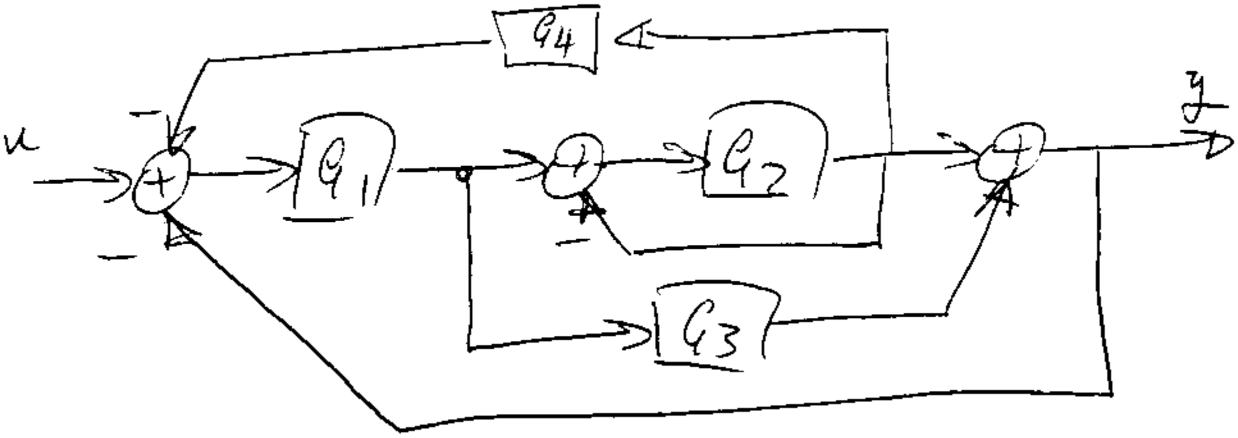




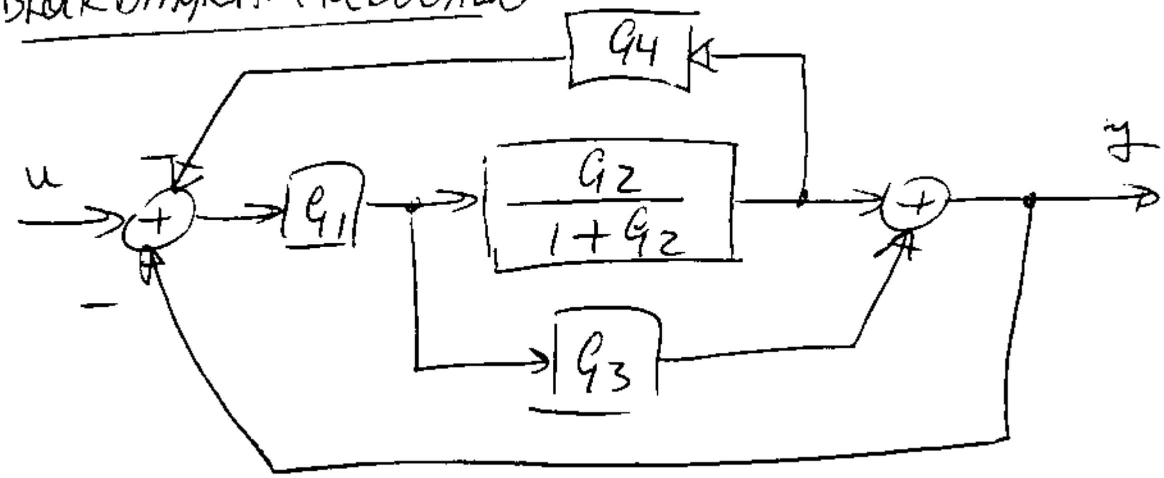
$$v_x = K_x x$$

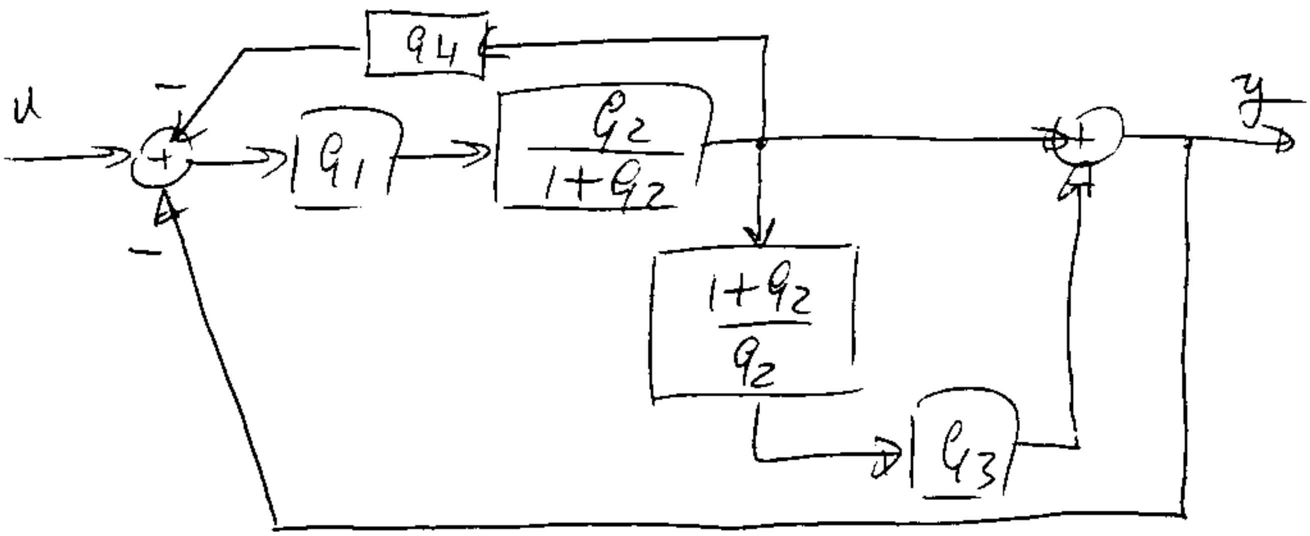

1/Rr
sC
-1
R
1/(Lf s+Rf)
Kf
1/(Js²+(N1/N2)²K1)
K2
Kx
x2 = x

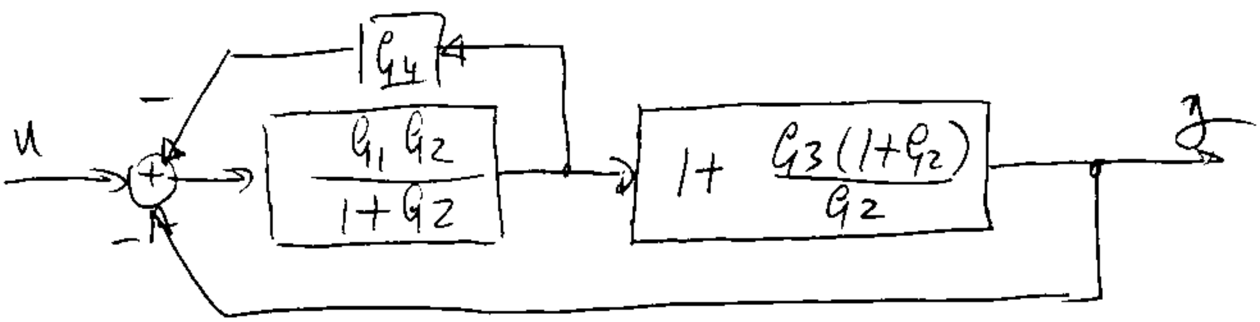
#3

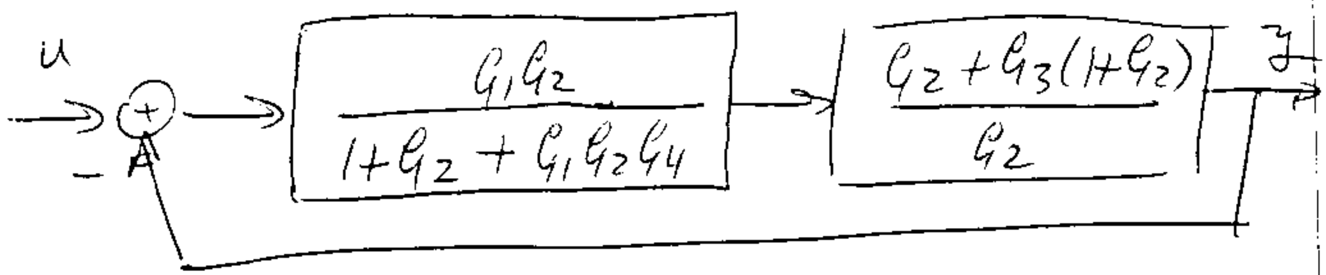


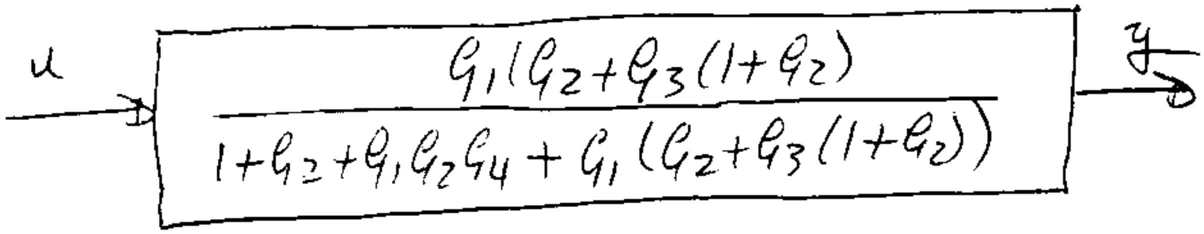
BLOCK DIAGRAM REDUCTION



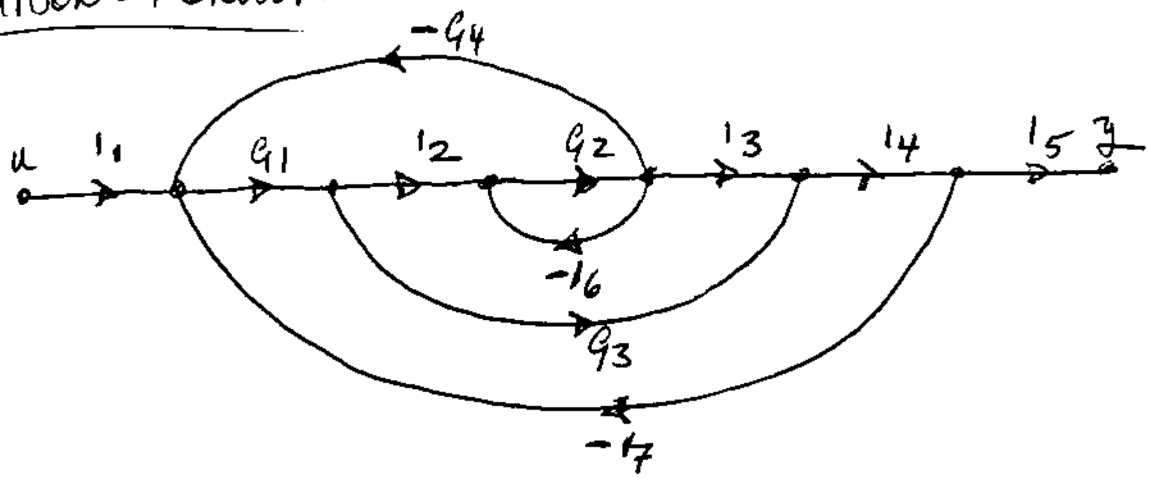








MASON'S FORMULA



FORWARD PATHS

$$F_1 = 1_1 G_1 1_2 G_2 1_3 1_4 1_5 = G_1 G_2$$
$$F_2 = 1_1 G_1 G_3 1_4 1_5 = G_1 G_3$$

LOOPS

$$L_1 = G_2(-1_6) = -G_2$$
$$L_2 = G_1 1_2 G_2(-G_4) = -G_1 G_2 G_4$$
$$L_3 = G_1 1_2 G_2 1_3 1_4(-1_7) = -G_1 G_2$$
$$L_4 = G_1 G_3 1_4(-1_7) = -G_1 G_3$$

TOUCHING LOOPS

	L_1	L_2	L_3	L_4
L_1		T	T	N
L_2			T	T
L_3				T
L_4				

Only non-touching pair is L_1 & L_4

LOOPS ON FORWARD PATHS

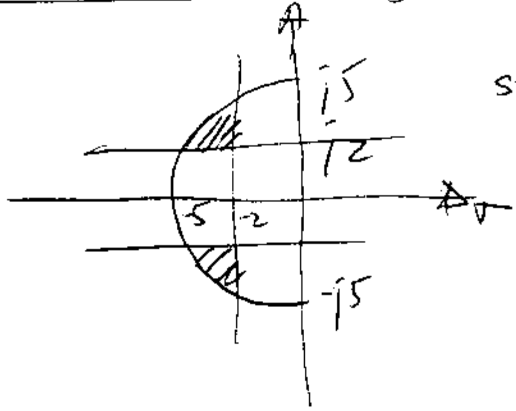
	L_1	L_2	L_3	L_4
F_1	O	O	O	O
F_2	N	O	O	O

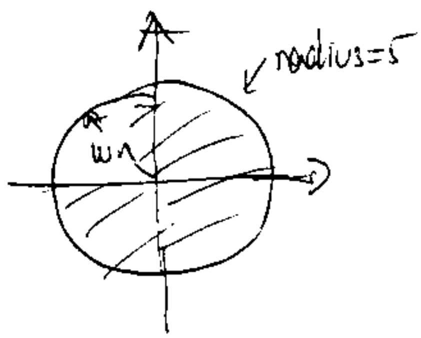
So
$$\Delta = 1-(L_1+L_2+L_3+L_4)+L_1L_4$$
$$= 1 + G_2 + G_1G_2G_4 + G_1G_2 + G_1G_3 + G_2G_1G_3$$

and
$$\Delta_1 = \Delta\Big|_{L_1=L_2=L_3=L_4=0} = 1$$
$$\Delta_2 = \Delta\Big|_{L_2=L_3=L_4=0} = 1-L_1 = 1+G_2$$

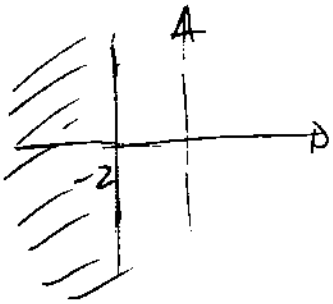
$$\Rightarrow \frac{Y}{U} = \frac{G_1G_2 + G_1G_3(1+G_2)}{1+G_2+G_1G_2G_4+G_1G_2+G_1G_3+G_1G_2G_3}$$

#4





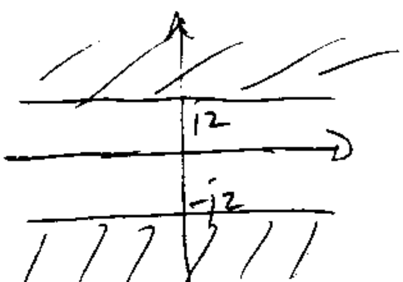
$$\boxed{\omega_n \le 5}$$



$$-\sigma_0 \le -2$$

$$\sigma_0 \ge 2$$

$$\boxed{\zeta \omega_n \ge 2}$$



$$j\omega_d \ge j2$$

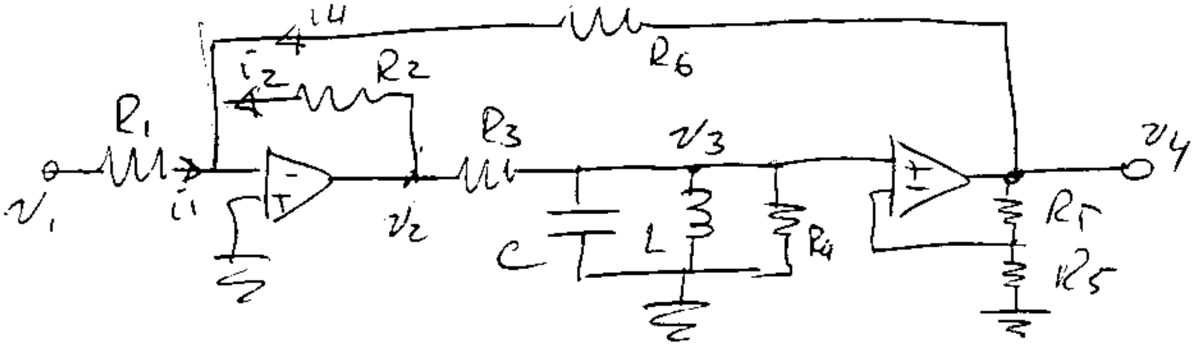
$$\omega_d \ge 2$$

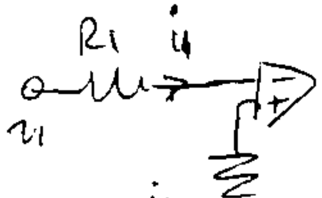
$$\boxed{\sqrt{1-\zeta^2}\,\omega_n \ge 2}$$

The desired region is the intersection $\Rightarrow$

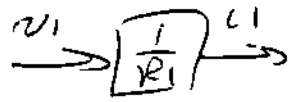
$$\boxed{\begin{array}{c} \omega_n \le 5 \\ \zeta\omega_n \ge 2 \\ \sqrt{1-\zeta^2}\,\omega_n \ge 2 \end{array}}$$

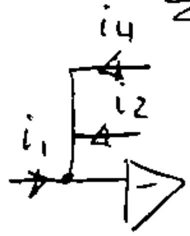
#5





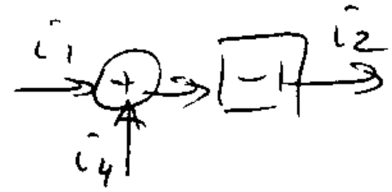
$$i_1 = \frac{1}{R_1} v_1$$

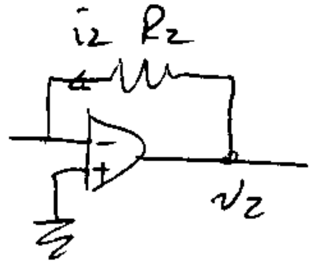




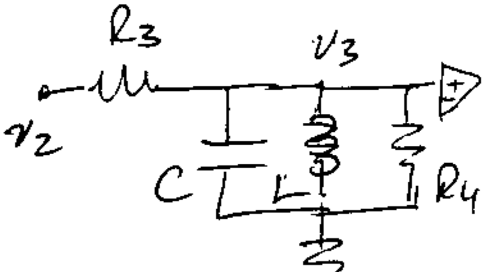
$$i_1 + i_2 + i_4 = 0$$

$$i_2 = -(i_1 + i_4)$$





$$v_2 = R_2 i_2$$

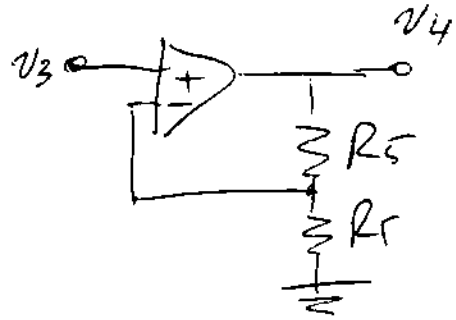


$$\frac{v_3}{\frac{1}{sC + \frac{1}{sL} + R_4}} = \frac{v_2}{R_3 + \frac{1}{sC + \frac{1}{sL} + R_4}}$$

$$v_3 = \frac{1}{R_3 C s + R_3/Ls + R_3 R_4 + 1} v_2$$

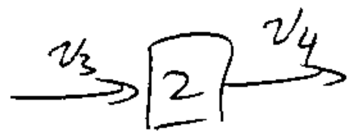
$$v_3 = \frac{s}{R_3 C s^2 + (1 + R_3 R_4) s + R_3/L} v_2$$

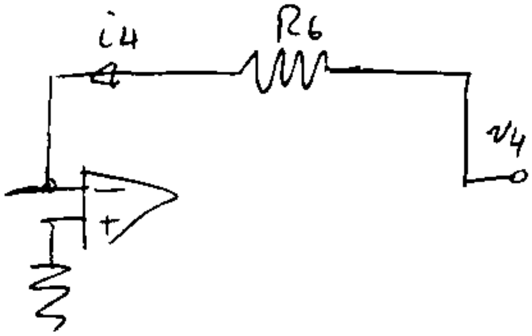
$$v_2 \longrightarrow \boxed{\frac{s}{R_3 C s^2 + (1 + R_3 R_4) s + R_3/L}} \longrightarrow v_3$$



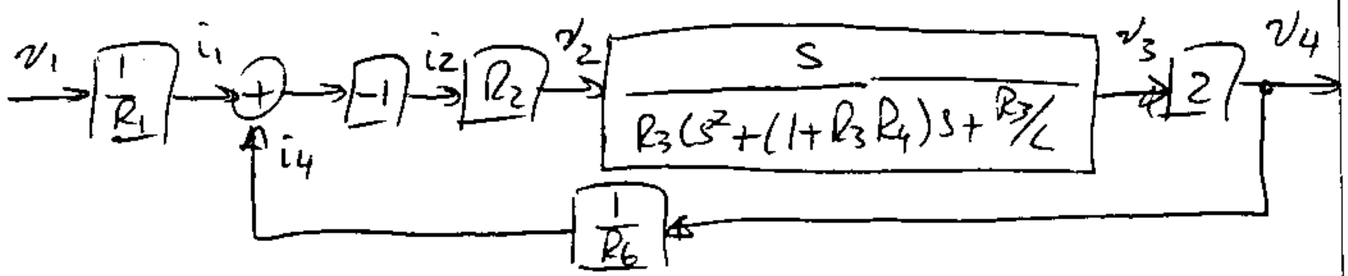
$$\frac{v_4}{2R_5} = \frac{v_3}{R_5}$$

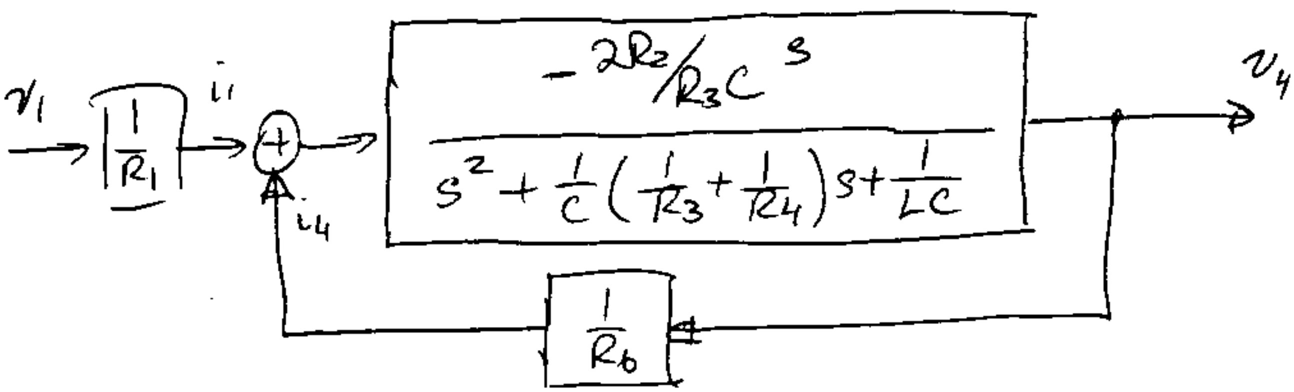
$$v_4 = 2\,v_3$$





$$i_4 = \frac{1}{R_6}\,v_4$$





$$\frac{v_4}{v_1} = \frac{1}{R1}\,\frac{-\frac{2R_2}{R_3C}\,s}{s^2 + \frac{1}{C}\left(\frac{1}{R_3}+\frac{1}{R_4}\right)s + \frac{1}{LC} - \left(-\frac{2R_2}{R_3C}s \cdot \frac{1}{R_6}\right)}$$

Closed-loop poles $\quad s^2 + \frac{1}{C}\left(\frac{1}{R_3}+\frac{1}{R_4}+\frac{2R_2}{R_3R_6}\right)s + \frac{1}{LC} = 0$

$$t_{5\%s} \approx \frac{40}{3} \Rightarrow \frac{3}{\zeta\omega_n} \simeq \frac{40}{3}$$

$$\zeta\omega_n = \frac{9}{40}$$

From the closed-loop pole eqn.

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

$$2\zeta\omega_n = \frac{1}{C}\left(\frac{1}{R_3}+\frac{1}{R_4}+\frac{2R_2}{R_3R_6}\right)$$

or $\quad \frac{1}{C}\left(\frac{1}{R_3}+\frac{1}{R_4}+\frac{2R_2}{R_3R_6}\right) = 2\cdot\frac{9}{40}$

$C = 1F$, $R_3 = 2\Omega$, $R_4 = 4\Omega$, and $R_6 = 5\Omega$

$$\frac{1}{1}\left(\frac{1}{2}+\frac{1}{4}+\frac{2R_2}{2\cdot 5}\right) = \frac{9}{20}$$

$$\frac{R_2}{5} = \frac{9}{20} - \frac{3}{4} = -\frac{6}{20}$$

$\Rightarrow R_2 = -\frac{3}{2}\,\Omega \quad$ Ops, this was supposed to be positive.