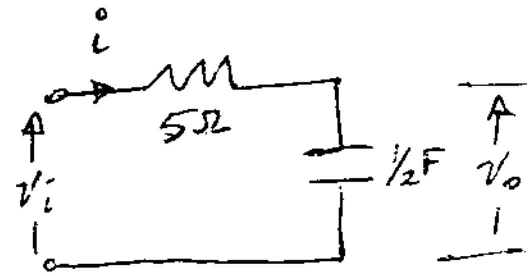
EE 231

Exam#2
75 minutes

Mar. 28, 1991

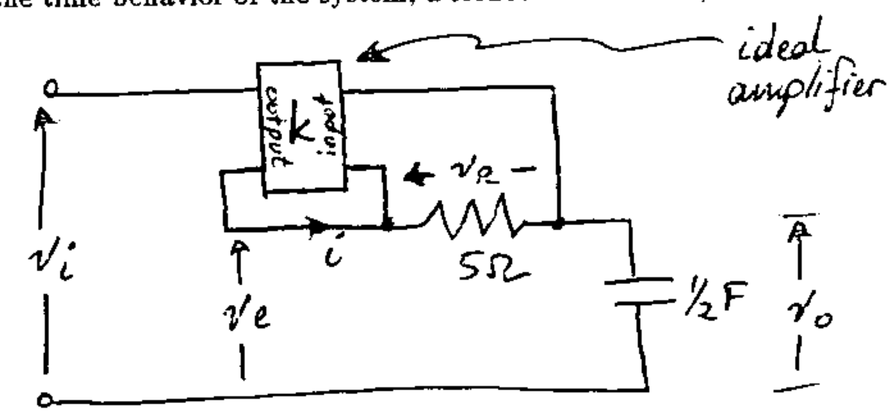
1. An RC-circuit is given below.



   (a) Calculate the time constant of this system. (5pts.)

   (b) Determine the 2% settling time for a unit-step input voltage. (5pts.)

   To improve the time behavior of the system, a feedback controller, as shown below, is utilized.



   (c) Determine the feedback proportionality constant k to reduce the time constant to 1/5th of its original value. *Note: Draw the block diagram of the circuit, such that variables v_i, i, v_o and v_e are clearly shown to see the effects of the controller.* (15pts.)

   (d) Find the sensitivity of the transfer function with respect to k for the k determined in part (1c) and for $\omega = 5\,\text{rad/sec}$. (10pts.)

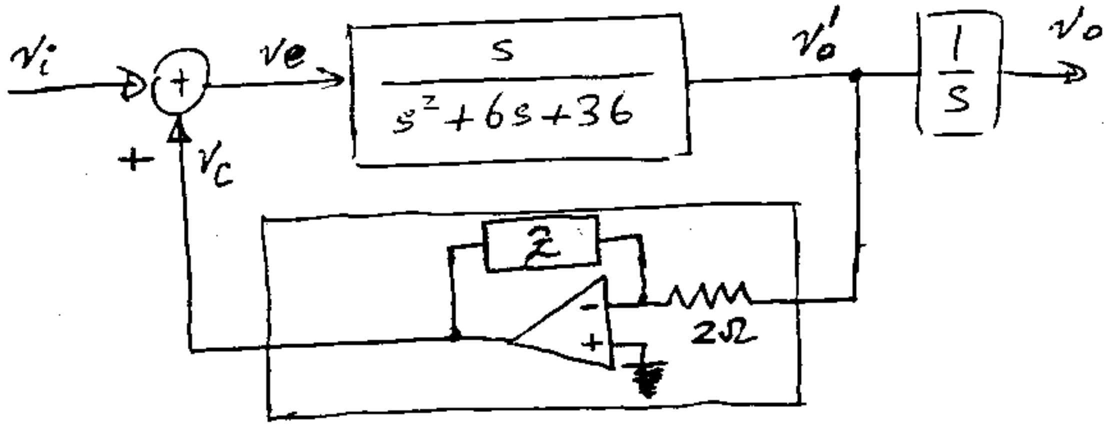
   (e) Determine the steady-state error voltage, $v_e(\infty)$ when the input voltage, $v_i(t)$ is a unit step. (5pts.)

2. An electrical circuit with an input voltage, v_i and an output voltage, v_0 has the following transfer function:

$$\frac{V_o(s)}{V_i(s)} = \frac{1}{s}\frac{s}{s^2+6s+36}\,.$$

   (a) Find the natural frequency and the damping constant of the system. (5pts)

To control the output voltage, the following feedback controller is implemented.



Here, Z is an unknown impedance.

(b) Pick a Z, such that a proportional controller is implemented, and the damping constant is 0.8. (15pts)
Note: Draw the block diagram to see the effects of the controller.

(c) Determine the steady-state error, $v_e(\infty)$ for unit-step and unit-ramp reference inputs. (10pts)

(d) To improve the steady-state behavior, this time pick another Z, such that a proportional-integral controller is implemented, and the natural frequency and the maximum percent overshoot are 8 and 1.5165%, respectively. (20pts)

(e) Determine the steady-state error for unit-step and unit-ramp reference inputs for the new case. (5pts)

3. A transfer function is given by

$$\frac{Y(s)}{U(s)} = \frac{10}{s} - \frac{1}{s^2+1} + \frac{2}{s^2+16} + \frac{5}{s+3} - \frac{2}{s+10} + \frac{1}{s+100}.$$

Determine the least-order approximation of the transfer function, such that the *approximation* error stays within 2% in 0.5 second for an impulse input. *Do not try to simplify the approximated transfer function.* (5pts)

#1 a) $T = 2.5$ sec

b) $t_{s\,2\%} = 10$ sec

c) $k = 0.8$

d) $S_k^{v_o/v_i}\Big|_{\omega = 5\,\text{rad/sec}} = 3.7139$

e) $v_e(\infty) = 1$

#2 a) $\omega_n = 6$, $\zeta = 0.5$

b) $Z = 7.2\,\Omega$

c) $v_e(\infty) = 1$ for unit step input

$v_e(\infty) = \infty$ for ramp input

d) $Z = 13.6 + \frac{56}{s}$ (Resistor & Capacitor)

e) $v_e(\infty) = 0.5625$ for unit step input

$v_e(\infty) = \infty$ for ramp input

#3 $$\frac{Y(s)}{U(s)} \approx \frac{10}{s} - \frac{1}{s^2+1} + \frac{2}{s^2+16} + \frac{5}{s+3}$$