EE 261 | Exam#3 | May 08, 1998

50 minutes

1. Sketch the asymptotes for the gain versus frequency plot for a system with the transfer function

$$G(s) = \frac{(s+100)^2}{s(s+1)}.$$

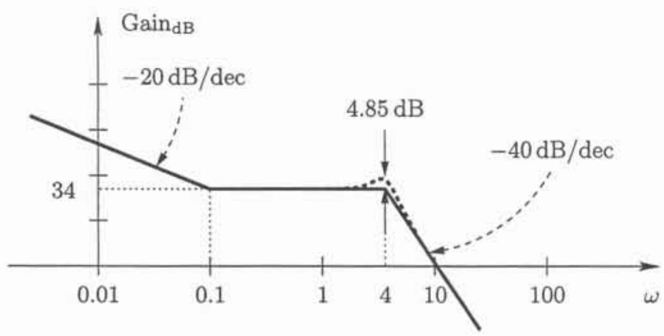
(15pts)

2. Determine the cut-off frequency or frequencies of a system with the transfer function

$$G(s) = \frac{10(s+1)}{(s+10)(s+500)}.$$

(15pts)

3. The frequency response of a minimum-phase, stable control system has been obtained experimentally, and the following asymptotes have been fitted.



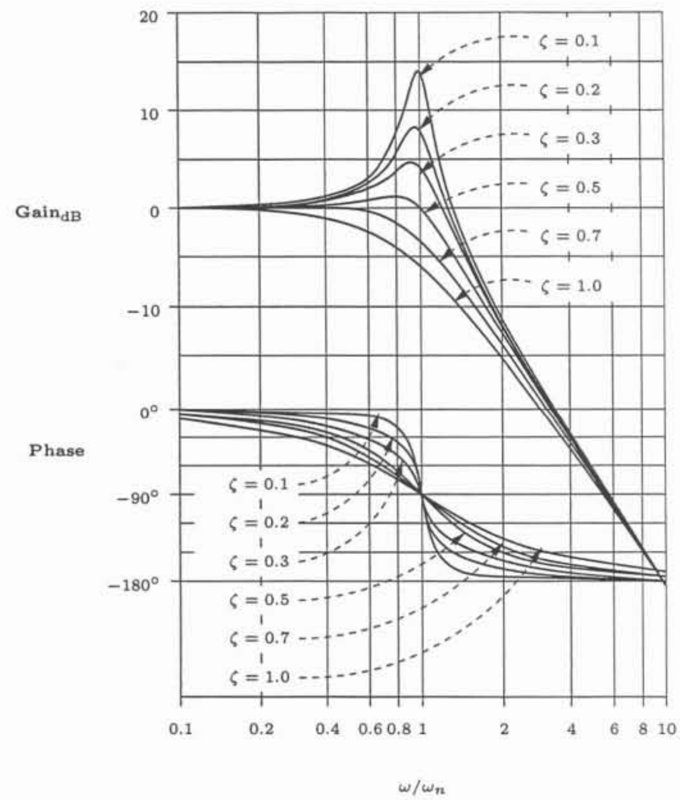
(a) Determine the transfer function from the asymptotes. (20pts)

(b) Sketch approximate phase versus frequency plot of the control system. (15pts)

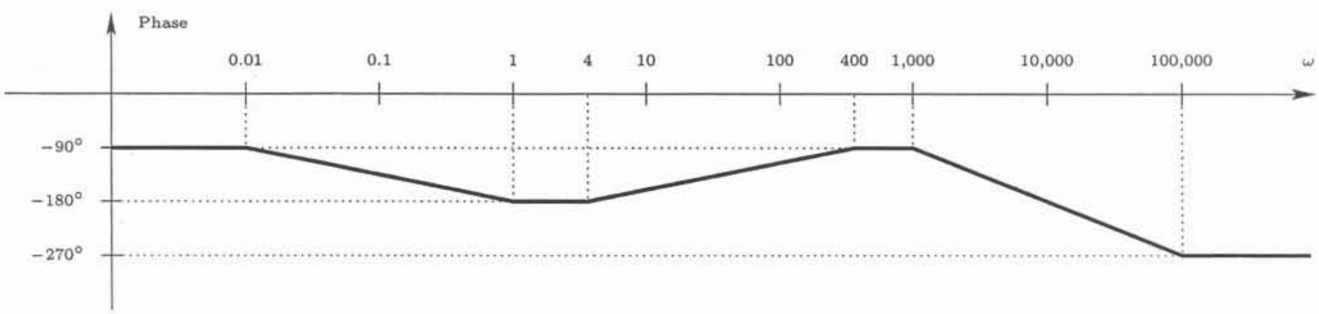
Make intelligent use of the figure on the right which shows the magnitude and phase versus frequency plots of

$$\frac{1}{(j(\omega/\omega_n))^2 + 2\zeta\,(j(\omega/\omega_n)) + 1}$$

for different values of ζ.



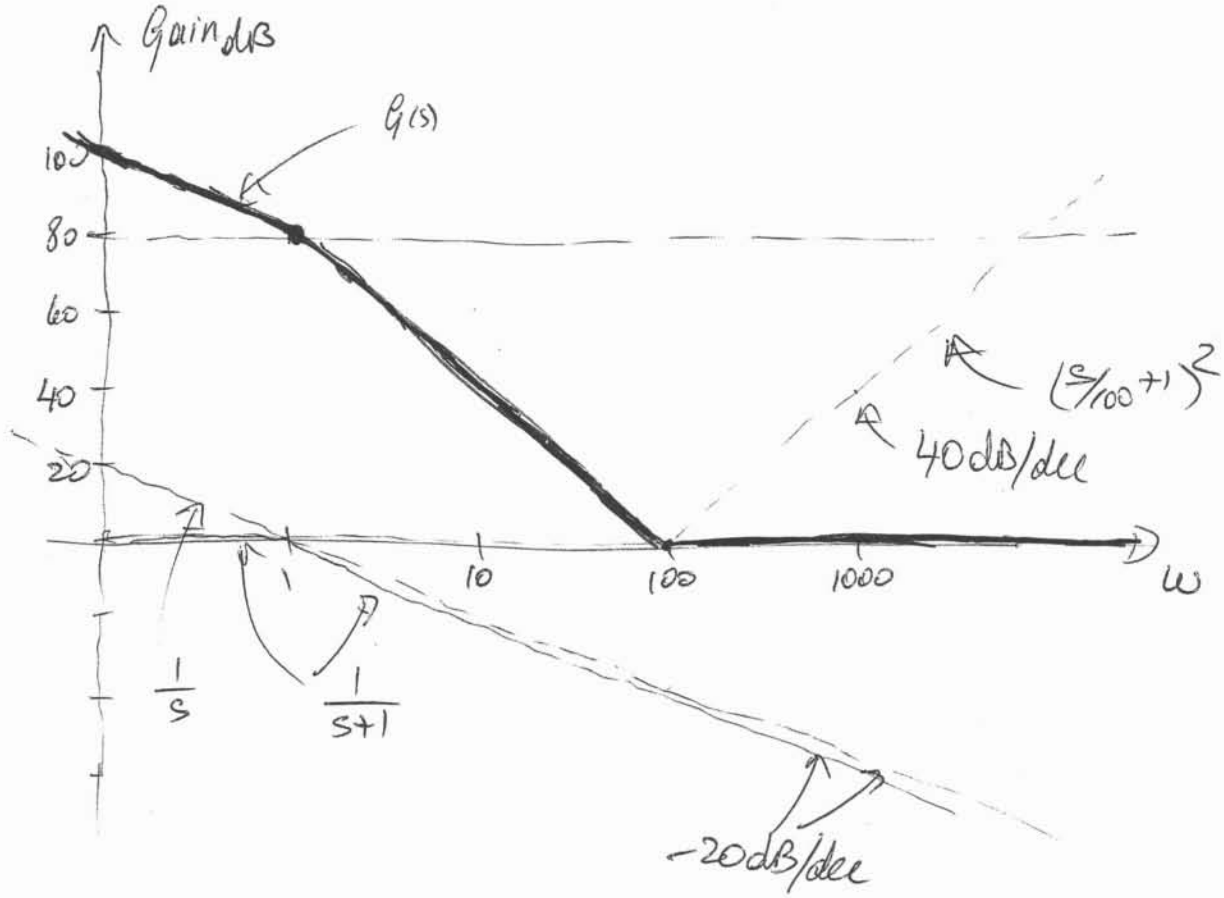
4. The phase of a certain minimal-phase, stable feedback control system has been obtained experimentally, and the following asymptotes were obtained.



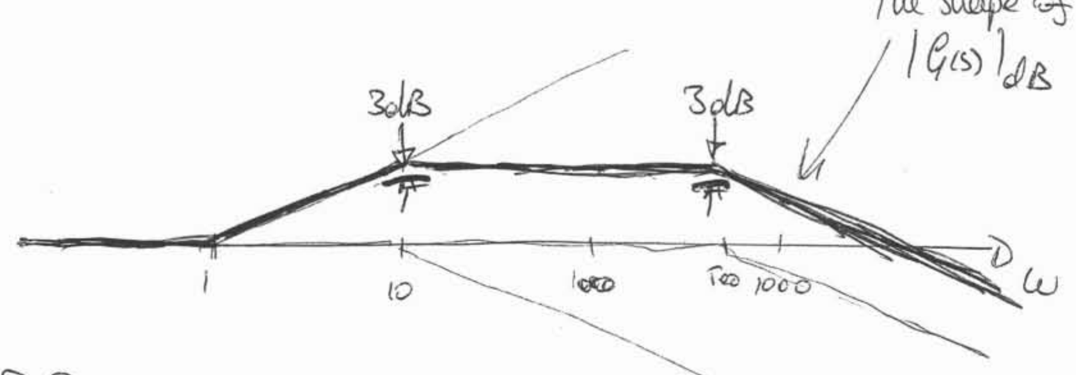
(a) Determine the transfer function from the asymptotes as much as possible, and leave the indeterminable quantities in parametric form. (20pts)

(b) Sketch approximate gain versus frequency plot assuming "reasonable" values for the missing quantities. Show your assumptions clearly. (15pts)

#1 $G(s) = \dfrac{(s+100)^2}{s(s+1)} = \dfrac{100^2 (s/100+1)^2}{s(s+1)}$



#2 $G(s) = \dfrac{10(s+1)}{(s+10)(s+500)}$

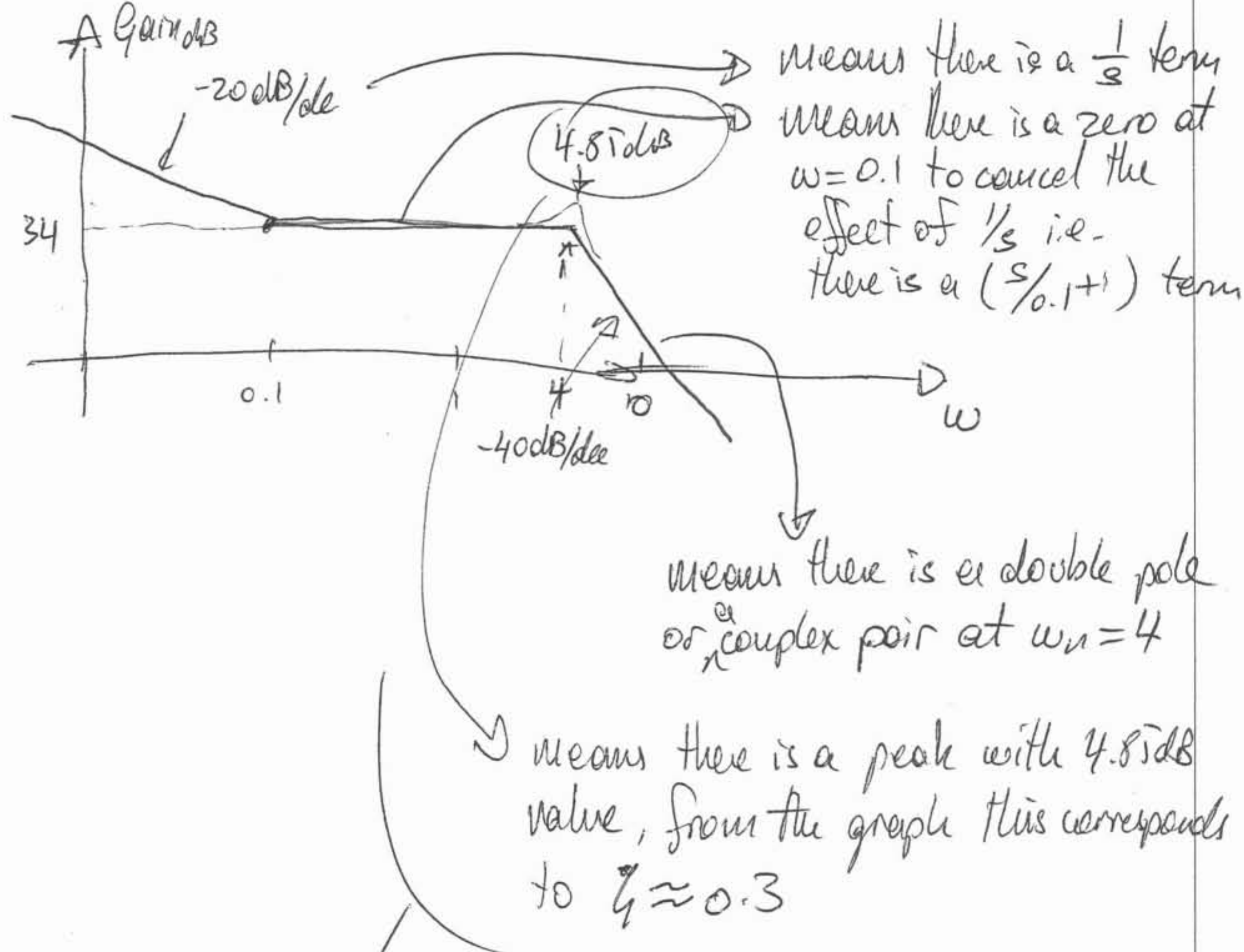


Cutoff freq when the gain is 3dB below the maximum since asymptotes are 3dB off at their change of direction locations.

in this case the cut-off frequencies are at

$\omega=10$ and $\omega=500$.

#3



Therefore, there is a complex pair at $\omega_n=4$ with $\zeta=0.3$ or

$$\frac{1}{\left(\frac{s}{4}\right)^2+2\times 0.3\times\left(\frac{s}{4}\right)+1}$$

So $$G(s)=K\,\frac{s/0.1+1}{s\left(\left(s/4\right)^2+0.6\left(s/4\right)+1\right)}$$

To determine K, we can use the magnitude at any frequency. However, the sketch, we have is only the asymptotes. So we either incorporate the difference between the asymptotes and the real plot, or pick a point where the asymptotes are almost the same as the real plot.

at $\omega = 0.1$ $\quad |G(s)|_{dB} = (34+3)\,dB = 37\,dB$

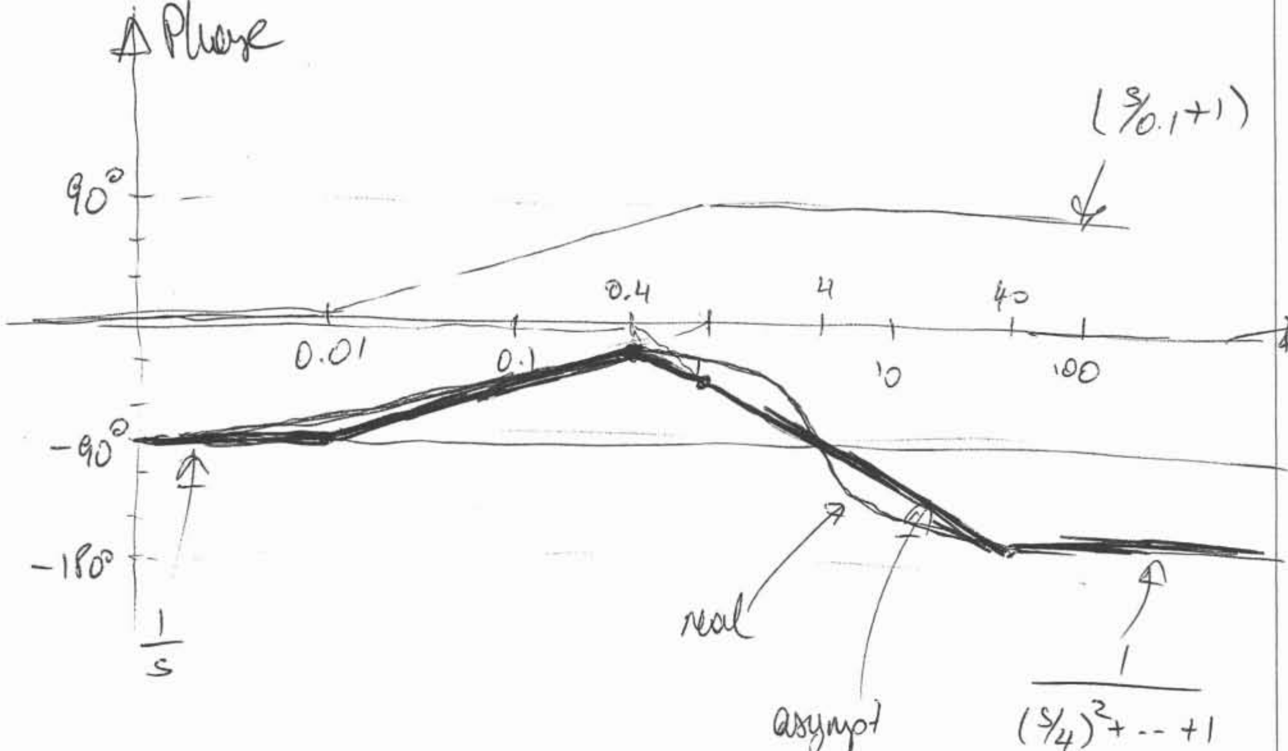
or $\quad |G(s)|_{s=j0.1} = 10^{37/20} = 70.8$

$$\left| K \frac{j0.1/0.1 + 1}{j0.1\left(\left(j0.1/4\right)^2 + 0.6\left(j0.1/4\right) + 1\right)} \right| = 70.8$$

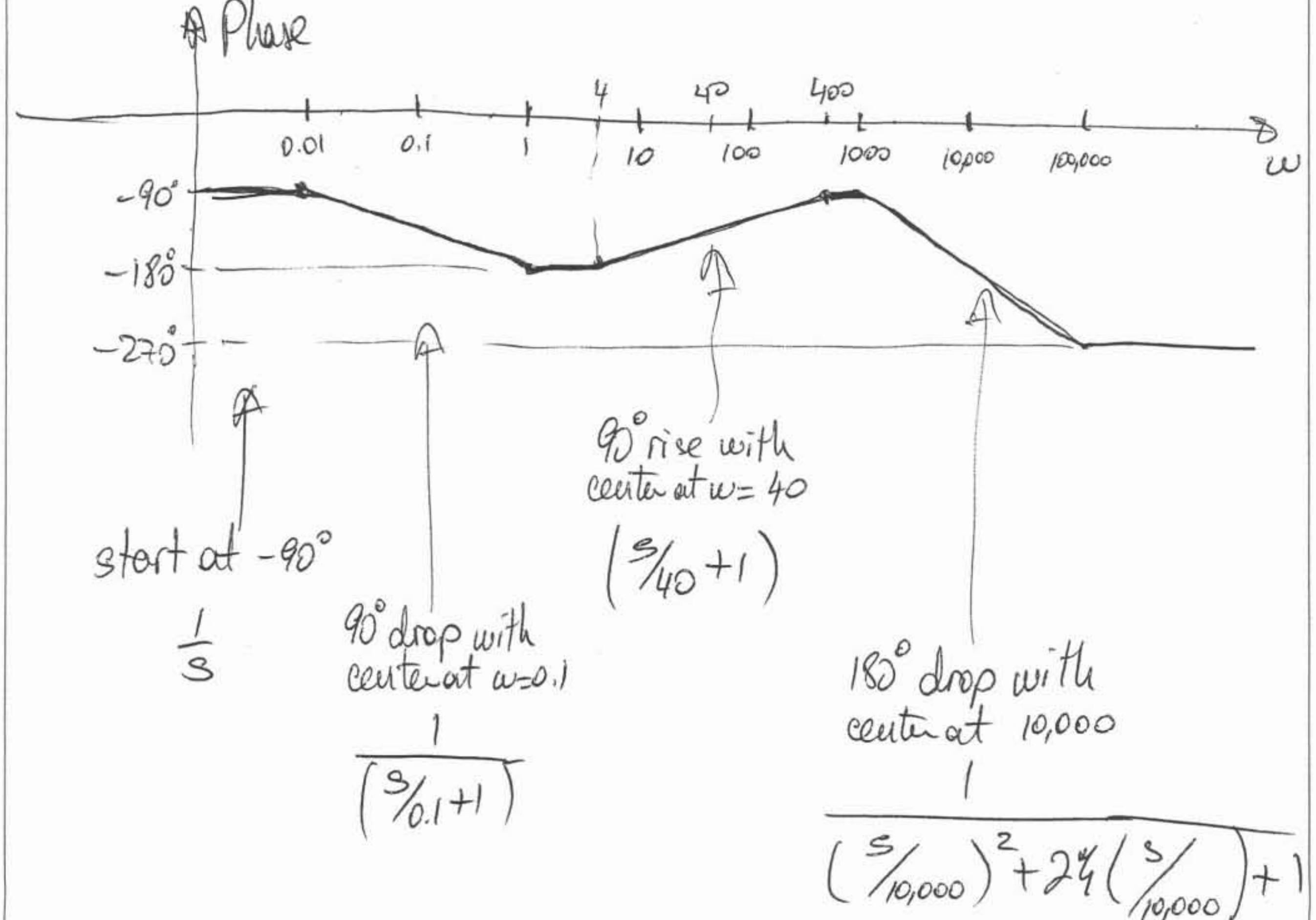
$\Rightarrow K \approx 5$

or $\quad G(s) = 5 \dfrac{s/0.1 + 1}{s\left(\left(s/4\right)^2 + 0.6\,(s/4) + 1\right)}$

$$= 800 \frac{s+0.1}{s(s^2+2.4s+16)}$$


Phase
90°
-90°
-180°
0.01
0.1
0.4
4
10
40
100
(s/0.1+1)
real
asympt
1/s
1/((s/4)^2 + ... + 1)

#4


Phase
ω
0.01
0.1
1
4
10
40
100
400
1000
10,000
100,000
-90°
-180°
-270°
start at -90°
1/s
90° drop with
center at ω=0.1
1/(s/0.1+1)
90° rise with
center at ω = 40
(s/40 + 1)
180° drop with
center at 10,000
1/((s/10,000)^2 + 2ζ(s/10,000) + 1)

so $G(s) = \dfrac{K\left(s/40 + 1\right)}{s\left(s/0.1 + 1\right)\left(\left(s/10{,}000\right)^2 + 2\zeta\left(s/10{,}000\right) + 1\right)}$

b) let $K = 1$, $\zeta = 1$ ← double pole