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1. A control system is described in state-space representation, such that

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} -3 & 0 & 2 \\ 0 & -1 & 0 \\ 2 & 0 & -3 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} u(t),$$

where \mathbf{x} and u are the state and the input variables, respectively.

- (a) Determine the poles of the system. (05pts)
- (b) Determine the linear transformation that would put it into the diagonal or Jordan form. Express the new state-space equation that describes the system. (20pts)
- (c) Obtain the solution $\mathbf{x}(t)$ for $t \geq 0$; when $\mathbf{x}(0) = [1 \ 0 \ 0]^T$, and $u(t) = 0$. (10pts)

2. Determine $\mathbf{x}(100)$ for the following discrete-time control systems, when $\mathbf{x}(0) = [1 \ 1]^T$.

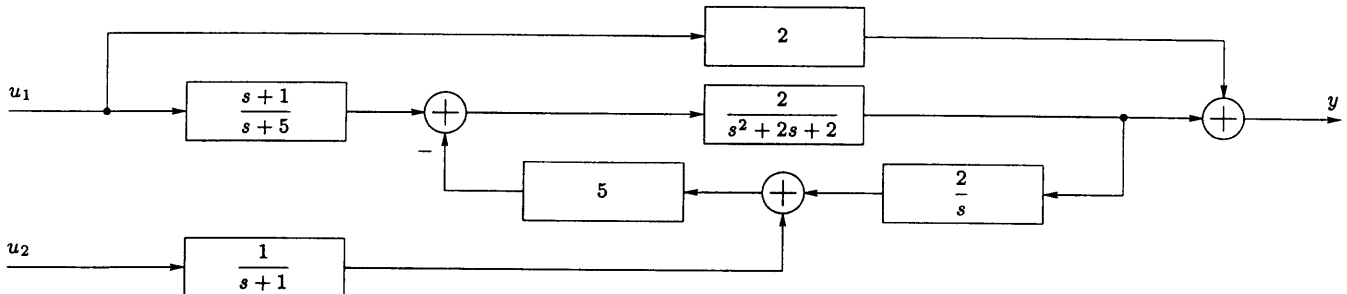
- (a) $\mathbf{x}(k+1) = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \mathbf{x}(k)$. (10pts)
- (b) $\mathbf{x}(k+1) = \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix} \mathbf{x}(k)$. (15pts)

3. A time-varying control system is described by

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} -e^{-t} & 0 \\ 0 & -1 - \cos(t) \end{bmatrix} \mathbf{x}(t),$$

where \mathbf{x} is the state variable. Determine $\mathbf{x}(5)$, when $\mathbf{x}(0) = [10 \ 20]^T$. (15pts)

4. The block diagram of a control system is given below.



Obtain a state-space representation of the system without any block-diagram reduction. (25pts)

#1

$$\dot{x} = \underbrace{\begin{bmatrix} -3 & 0 & 2 \\ 0 & -1 & 0 \\ 2 & 0 & -3 \end{bmatrix}}_A x + \underbrace{\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}}_B u$$

(a) Poles or the eigenvalues from the solution of the characteristic equation.

$$\det(\lambda I - A) = 0$$

$$\det \begin{bmatrix} \lambda+3 & 0 & -2 \\ 0 & \lambda+1 & 0 \\ -2 & 0 & \lambda+3 \end{bmatrix} = 0$$

$$[(\lambda+3)(\lambda+1)(\lambda+3) + 0 + 0] - [(-2)(\lambda+1)(-2) + 0 + 0] = 0$$

$$(\lambda+1)[(\lambda+3)^2 - 4] = 0$$

$$(\lambda+1)(\lambda^2 + 6\lambda + 5) = 0$$

$$(\lambda+1)(\lambda+1)(\lambda+5) = 0 \Rightarrow \lambda = -1, -1, -5$$

(b) The transformation $x = Tz$ is when

$$T = \begin{bmatrix} u_1 & u_2 & u_3 \\ \downarrow & \downarrow & \downarrow \end{bmatrix} \text{ where } u_i \text{'s are the eigenvectors}$$

Moreover, since A is symmetric, T can be chosen orthonormal.

$$(A - \lambda I)u = 0$$

$$\text{or } (\lambda I - A)u = 0 \Rightarrow \begin{bmatrix} \lambda+3 & 0 & -2 \\ 0 & \lambda+1 & 0 \\ -2 & 0 & \lambda+3 \end{bmatrix} \begin{bmatrix} u_{1i} \\ u_{2i} \\ u_{3i} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\lambda = -1 \Rightarrow \begin{bmatrix} 2 & 0 & -2 \\ 0 & 0 & 0 \\ -2 & 0 & 2 \end{bmatrix} \begin{bmatrix} u_{11} \\ u_{21} \\ u_{31} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$2u_{11} - 2u_{31} = 0$ is the only eqn.

$$\Rightarrow u_{11} = u_{31}$$

let $u_{11} = 1 \Rightarrow u_{31} = 1 \Rightarrow u_1 = \begin{bmatrix} 1 \\ a \\ 1 \end{bmatrix}$

another choice let $u_{11} = 0 \Rightarrow u_{31} = 0 \Rightarrow u_2 = \begin{bmatrix} 0 \\ b \\ 0 \end{bmatrix}$

Choose $\langle u_1, u_2 \rangle = 0$

$$\langle u_1, u_2 \rangle = (1)(0) + a \cdot b + (1)(0)$$

$$= ab$$

$$\Rightarrow a = 0 \text{ or } b = 0$$

since $b = 0 \Rightarrow u_2 = 0$ not a valid eigenvector

so $a = 0$ and let $b = 1$

$$\Rightarrow u_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \xrightarrow{\text{normalize}} u_1^* = \begin{bmatrix} \sqrt{2}/2 \\ 0 \\ \sqrt{2}/2 \end{bmatrix}$$

$$u_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \longrightarrow u_2^* = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\lambda = -5 \Rightarrow \begin{bmatrix} -2 & 0 & -2 \\ 0 & -4 & 0 \\ -2 & 0 & -2 \end{bmatrix} \begin{bmatrix} u_{13} \\ u_{23} \\ u_{33} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-2u_{13} - 2u_{33} = 0 \Rightarrow u_{33} = -u_{13}$$

$$-4u_{23} = 0 \Rightarrow u_{23} = 0$$

$$\text{let } u_{13} = 1 \Rightarrow u_3 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \rightarrow u_3^* = \begin{bmatrix} \sqrt{2}/2 \\ 0 \\ -\sqrt{2}/2 \end{bmatrix}$$

$$\text{or } T = \begin{bmatrix} \sqrt{2}/2 & 0 & \sqrt{2}/2 \\ 0 & 1 & 0 \\ \sqrt{2}/2 & 0 & -\sqrt{2}/2 \end{bmatrix}$$

$$\begin{aligned} \text{let } x = Tz &\Rightarrow \dot{z} = T^T A T z + T^T B u \\ &= \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -5 \end{bmatrix} z + \begin{bmatrix} \sqrt{2}/2 & 0 & \sqrt{2}/2 \\ 0 & 1 & 0 \\ \sqrt{2}/2 & 0 & -\sqrt{2}/2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} u \\ \dot{z} &= \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -5 \end{bmatrix} z + \begin{bmatrix} 0 \\ 0 \\ \sqrt{2} \end{bmatrix} u \end{aligned}$$

$$\textcircled{c} \quad x(t) = e^{At} x(0) \quad \text{when } u(t) = 0$$

$$\begin{aligned} &= T e^{T^T A T t} T^t x(0) \\ &= T e^{\Lambda t} T^t x(0) \\ &= \begin{bmatrix} \sqrt{2}/2 & 0 & \sqrt{2}/2 \\ 0 & 1 & 0 \\ \sqrt{2}/2 & 0 & -\sqrt{2}/2 \end{bmatrix} \begin{bmatrix} e^{-t} & & \\ & e^{-t} & \\ & & e^{-5t} \end{bmatrix} \begin{bmatrix} \sqrt{2}/2 & 0 & \sqrt{2}/2 \\ 0 & 1 & 0 \\ \sqrt{2}/2 & 0 & -\sqrt{2}/2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \\ &= \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \begin{bmatrix} e^{-t} & & \\ & e^{-t} & \\ & & e^{-5t} \end{bmatrix} \begin{bmatrix} \sqrt{2}/2 \\ 0 \\ \sqrt{2}/2 \end{bmatrix} \\ &= \begin{bmatrix} \sqrt{2}/2 & 0 & \sqrt{2}/2 \\ 0 & 1 & 0 \\ \sqrt{2}/2 & 0 & -\sqrt{2}/2 \end{bmatrix} \begin{bmatrix} \sqrt{2}/2 e^{-t} \\ 0 \\ \sqrt{2}/2 e^{-5t} \end{bmatrix} = \begin{bmatrix} 1/2 e^{-t} + 1/2 e^{-5t} \\ 0 \\ 1/2 e^{-t} - 1/2 e^{-5t} \end{bmatrix} \end{aligned}$$

$$\#2 \quad x(k+1) = Ax(k) \Rightarrow x(k) = A^k x(0)$$

$$x(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$(a) \quad A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\begin{aligned} x(100) &= A^{100} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} (-1)^{100} & 0 \\ 0 & (-1)^{100} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \end{aligned}$$

$$(b) \quad A = \begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix} \quad \leftarrow \lambda = -1, -1 \text{ since upper diagonal}$$

$$x(100) = A^{100} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$A^{100} = aI + bA \quad \text{and} \quad \lambda^{100} = a + b\lambda$$

$$\frac{d}{d\lambda} (\lambda^{100} = a + b\lambda) \quad \text{since } \lambda \text{ is repeated}$$

$$\text{or} \quad \left. \begin{aligned} \lambda^{100} &= a + b\lambda \\ 100 \lambda^{99} &= b \end{aligned} \right\} \lambda = -1 \quad \left\{ \begin{aligned} (-1)^{100} &= a + b(-1) \\ 100(-1)^{99} &= b \end{aligned} \right.$$

$$\Rightarrow b = -100 \Rightarrow a = b + 1 = -99$$

$$A^{100} = -99I - 100A$$

$$= \begin{bmatrix} -99 & 0 \\ 0 & -99 \end{bmatrix} + \begin{bmatrix} 100 & -100 \\ 0 & 100 \end{bmatrix} = \begin{bmatrix} 1 & -100 \\ 0 & 1 \end{bmatrix}$$

$$x(100) = \begin{bmatrix} 1 & -100 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -99 \\ 1 \end{bmatrix}$$

43-282 500 SHEETS FULLER 5 SQUARE
42-301 50 SHEETS EYE-GLASS 5 SQUARE
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42-303 100 SHEETS EYE-GLASS 5 SQUARE
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#3

$$\dot{x} = \begin{bmatrix} -e^{-t} & 0 \\ 0 & -1 - \cos t \end{bmatrix} x, \quad x(0) = \begin{bmatrix} 10 \\ 20 \end{bmatrix}$$

$$x(t) = e^{\int_0^t A(z) dz} \cdot x(0)$$

$$= e^{\begin{bmatrix} -\int_0^t e^{-z} dz & 0 \\ 0 & -\int_0^t (1 + \cos z) dz \end{bmatrix}} \begin{bmatrix} 10 \\ 20 \end{bmatrix}$$

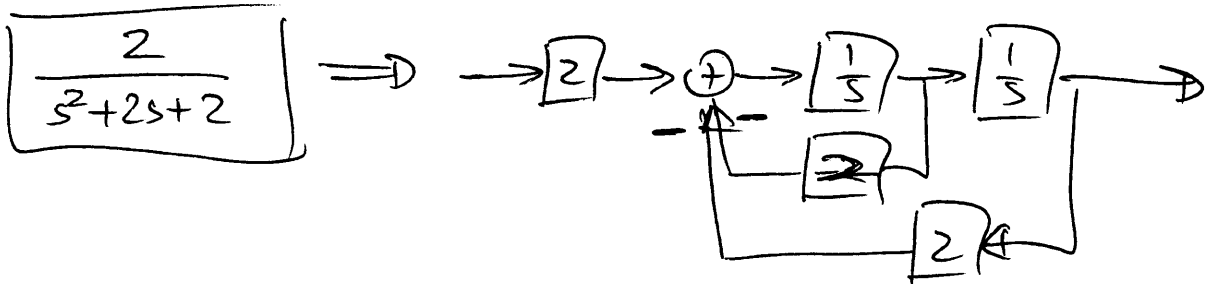
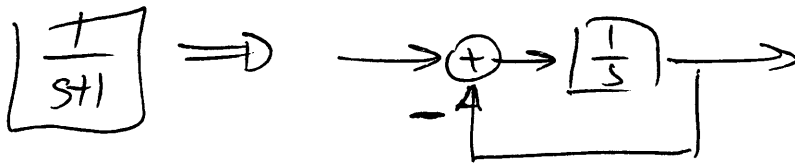
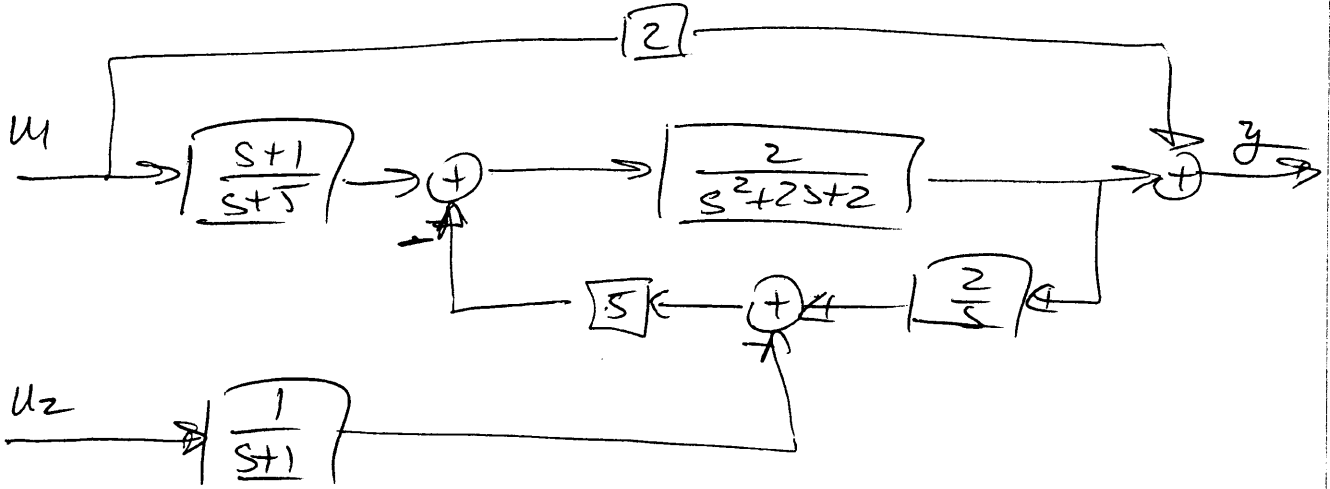
$$= e^{\begin{bmatrix} e^{-t} - 1 & 0 \\ 0 & -(t + \sin t) + (0 + 0) \end{bmatrix}} \begin{bmatrix} 10 \\ 20 \end{bmatrix}$$

$$= \begin{bmatrix} e^{e^{-t} - 1} & 0 \\ 0 & e^{-t - \sin t} \end{bmatrix} \begin{bmatrix} 10 \\ 20 \end{bmatrix} = \begin{bmatrix} 10 e^{e^{-t} - 1} \\ 20 e^{-t - \sin t} \end{bmatrix}$$

$$x(5) = \begin{bmatrix} 10 e^{e^{-5} - 1} \\ 20 e^{-5 - \sin 5} \end{bmatrix} = \begin{bmatrix} 3.7037 \\ 0.3516 \end{bmatrix}$$

in radium

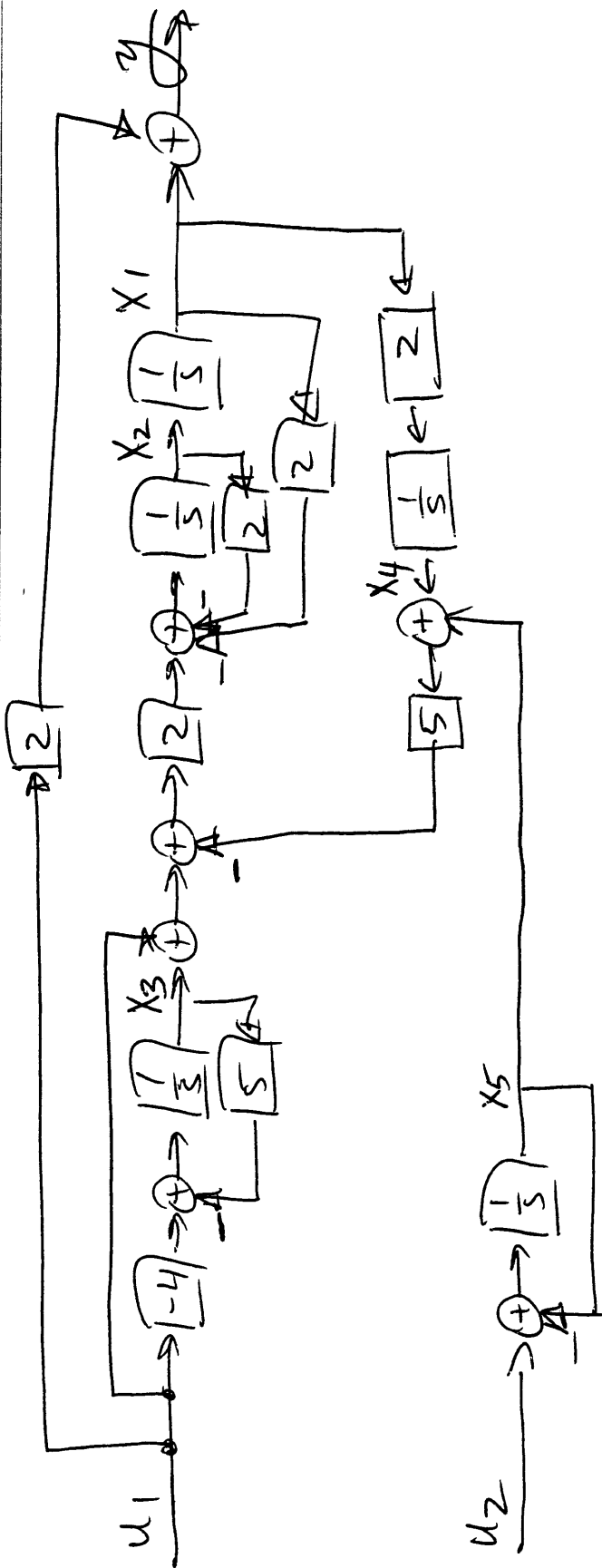
#4



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13-7932 500 SHEETS, FILLER, 5 SQUARE
42-381 50 SHEETS EYE-EASE, 5 SQUARE
42-382 100 SHEETS EYE-EASE, 5 SQUARE
42-392 100 SHEETS EYE-EASE, 5 SQUARE
42-393 100 RECYCLED WHITE, 5 SQUARE
42-399 200 RECYCLED WHITE, 5 SQUARE
Made in U.S.A.



$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -2x_2 - 2x_1 + 2[(u_1 + x_3) - 5(x_4 + x_5)]$$

$$= -2x_1 - 2x_2 + 2x_3 - 10x_4 - 10x_5 + 2u_1$$

$$\dot{x}_3 = -5x_3 - 4u_1$$

$$\dot{x}_4 = 2x_1$$

$$\dot{x}_5 = -x_5 + u_2$$

$$y = x_1 + 2u_1$$

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ -2 & -2 & 2 & -10 & -10 \\ 0 & 0 & -5 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix} x + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 2 & -4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$y = [1 \ 0 \ 0 \ 0 \ 0] x + [2 \ 0] \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$