Math 5222 Lecture 15 Problems

Problems

1. Show that $R^{\alpha}_{\alpha jk} \equiv 0$.

2. If $R_{ij} = \rho g_{ij}$, then $\rho = R/n$, where $R = g^{ij}R_{ij}$. (The equation $R_{ij} = \rho g_{ij}$ is known as the Einstein gravitational equation at points where matter is present. It corresponds to the Poisson equation $\nabla^2 V = \rho$ in the Newtonian theory of gravitation.)

3. If $n = 2$, show that $R_{11}/g_{11} = R_{22}/g_{22} = R_{12}/g_{12} = -R_{1212}/g$.

4. If $n = 3$, the tensor R_{ijkl} has six distinct components, and there are six equations $R_{jk} = g^{il}R_{ijkl}$. Prove that the solutions of these equations for R_{ijkl} are given by

$$R_{ijkl} = g_{il}R_{jk} + g_{jk}R_{il} - g_{ik}R_{jl} - g_{jl}R_{il} + \frac{R}{2}(g_{ik}g_{jl} - g_{il}g_{jk}),$$

where $R = g^{ij}R_{ij}$.

5. Verify Bianchi's identity 38.2.

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