

Problems from Math 5222 Lecture 3

Problems

- ✓1. Show that if the transformation $T: y^i = a_j^i x^j$ is orthogonal, then the distinction between the covariant and contravariant laws disappears.
- ✓2. Prove the theorem: If $f(x^1, x^2, \dots, x^n)$ is a homogeneous function of degree m , then $\frac{\partial f}{\partial x^i} x^i = mf$.
3. Given $f(x^1, x^2, \dots, x^n)$ and a set of equations of transformation $x^i = x^i(y^1, y^2, \dots, y^n)$, where each $y^i = y^i(t)$. If the transform of f by invariance is $g(y^1, y^2, \dots, y^n)$, show that $df/dt = dg/dt$. Hint: $(\partial f / \partial x^\alpha)(dx^\alpha/dt) = df/dt$ and $dx^\alpha/dt = (\partial x^\alpha / \partial y^j)(dy^j/dt)$.
- ✓4. Write out the laws of transformation of components of covariant and contravariant vectors when T is the transformation from rectangular cartesian to spherical polar coordinates given in Sec. 19.

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