

Problems from Math 5222 Lecture 7

Problems

1. Let E_3 be covered by orthogonal cartesian coordinates x^i , and consider a transformation

$$x^1 = y^1 \sin y^2 \cos y^3,$$

$$x^2 = y^1 \sin y^2 \sin y^3,$$

$$x^3 = y^1 \cos y^2,$$

where the y^i are spherical polar coordinates ($y^1 = r, y^2 = \theta, y^3 = \phi$). What are the metric coefficients $g_{ij}(y)$?

2. Let E_3 be covered by orthogonal cartesian coordinates x^i , and let

$$x^1 = y^1 \cos y^2,$$

$$x^2 = y^1 \sin y^2,$$

$$x^3 = y^3$$

represent a transformation to cylindrical coordinates y^i . Find the expression for ds^2 in cylindrical coordinates.

3. Let E_3 be covered by orthogonal cartesian coordinates x^i , and let $x^i = a_j^i y^j$, $|a_j^i| \neq 0$, ($i, j = 1, 2, 3$), represent a linear transformation of coordinates. Determine the metric coefficients $g_{ij}(y)$. Discuss the case when the transformation is orthogonal.