Problems from Math 5222 Lecture 9

Problems

✓**1.** Show that $\frac{\partial g_{ij}}{\partial x^k} - \frac{\partial g_{jk}}{\partial x^i} = [jk, i] - [ij, k]$.

✓**2.** Show that, if $g_{ij} = 0$ for $i \neq j$, then $\begin{Bmatrix} k \\ ij \end{Bmatrix} = 0$ whenever i, j, and k are distinct.

✓**3.** Show that, if $g_{ij} = 0$ for $i \neq j$, then

$$\begin{Bmatrix} i \\ ii \end{Bmatrix} = \frac{1}{2}\frac{\partial}{\partial x^i}\log g_{ii}, \qquad \begin{Bmatrix} i \\ ij \end{Bmatrix} = \frac{1}{2}\frac{\partial}{\partial x^j}\log g_{ii}, \qquad \begin{Bmatrix} i \\ jj \end{Bmatrix} = -\frac{1}{2g_{ii}}\frac{\partial g_{jj}}{\partial x^i},$$

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where we suspend the summation convention and suppose that $i \neq j$.

✓ **4.** If $|g_{ij}| \neq 0$, show that

$$g_{\alpha\beta}\frac{\partial}{\partial x^j}\begin{Bmatrix} \beta \\ ik \end{Bmatrix} = \frac{\partial}{\partial x^j}[ik, \alpha] - \begin{Bmatrix} \beta \\ ik \end{Bmatrix}([\beta j, \alpha] + [\alpha j, \beta]).$$

5. If $y^i = a_j{}^i x^j$ is a transformation from a set of orthogonal cartesian variables y^i to a set of oblique cartesian coordinates x^i covering E_3, what are the metric coefficients g_{ij} in $ds^2 = g_{ij}\,dx^i\,dx^j$?