

Mathematics 325  
Homework 10

Due Date: \_\_\_\_\_

Name: \_\_\_\_\_

Work exercise 1 on page 54 of Strauss.

1. Show that there is no maximum principle for the wave equation.

Consider  $u(x,t) = \sin(x)\sin(ct)$  on the rectangle  $\bar{R}: 0 \leq x \leq \pi, 0 \leq t \leq \pi/c$

First,  $u_{tt} - c^2 u_{xx} = -c^2 \sin(x)\sin(ct) - c^2[-\sin(x)\sin(ct)] = 0$  so  $u$  solves the wave equation on  $\bar{R}$ . Next, note that for any point  $(x_0, t_0)$  on the boundary  $\partial\bar{R}$  of  $\bar{R}$  we have  $u(x_0, t_0) = 0$ . Finally, observe that  $u(\pi/2, \pi/2c) = \sin(\pi/2)\sin(c \cdot \pi/2c) = 1$  so

$$\max_{(x,t) \in \partial\bar{R}} u(x,t) = 0 < 1 = \max_{(x,t) \in \bar{R}} u(x,t).$$