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2. $f(x) = x^2 - 1$. $\quad f'(x) = \lim_{h\to 0} \dfrac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \dfrac{(x+h)^2 - 1 - (x^2-1)}{h}$

$$= \lim_{h\to 0} \frac{x^2+2xh+h^2-1-x^2+1}{h}$$

$$= \lim_{h\to 0} \frac{2xh+h^2}{h} = \lim_{h\to 0} (2x+h) = 2x$$

slope of tangent line at $x = -1$ is $m = f'(-1) = -2$.

6. $f(x) = \dfrac{1}{x^2}$ $\quad f'(x) = \lim_{h\to 0} \dfrac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \dfrac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h}$

$$= \lim_{h\to 0} \frac{\frac{x^2}{x^2(x+h)^2} - \frac{(x+h)^2}{x^2(x+h)^2}}{\frac{h}{1}}$$

$$= \lim_{h\to 0} \frac{(x^2 - (x+h)^2)\cdot 1}{x^2(x+h)^2 \cdot h} = \lim_{h\to 0} \frac{x^2 - x^2 - 2xh - h^2}{x^2(x+h)^2 \cdot h}$$

$$= \lim_{h\to 0} \frac{-2xh - h^2}{hx^2(x+h)^2} = \lim_{h\to 0} \frac{-2x-h}{x^2(x+h)^2} = \frac{-2x}{x^4} = -2x^{-3}$$

Slope of tangent line at $x = 2$ is $m = f'(2) = \dfrac{-2}{8} = \dfrac{-1}{4}$

10. $f(x) = x^3 - x$ $\quad f'(x) = \lim_{h\to 0} \dfrac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \dfrac{[(x+h)^3 - (x+h)] - (x^3-x)}{h}$

$$= \lim_{h\to 0} \frac{x^3+3x^2h+3xh^2+h^3-x-h-x^3+x}{h}$$

$$= \lim_{h\to 0} \frac{3x^2h+3xh^2+h^3-h}{h} = \lim_{h\to 0} (3x^2+3xh+h^2-1)$$

$$= 3x^2 - 1$$

If $x_0 = -2$, then $y_0 = f(x_0) = f(-2) = (-2)^3 - (-2) = -6$. point $(-2, -6)$

$m = f'(-2) = 3(-2)^2 - 1 = 11$

Line: $y + 6 = 11(x+2)$, or $y = 11x - 16$.

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16. $y = \sqrt{1-x}$

$$\frac{dy}{dx} = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\sqrt{1-(x+h)} - \sqrt{1-x}}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{1-x-h} - \sqrt{1-x}}{h} \cdot \frac{\sqrt{1-x-h} + \sqrt{1-x}}{\sqrt{1-x-h} + \sqrt{1-x}}$$

$$= \lim_{h\to 0} \frac{(1-x-h)-(1-x)}{h(\sqrt{1-x-h}+\sqrt{1-x})} = \lim_{h\to 0} \frac{-h}{h(\sqrt{1-x-h}+\sqrt{1-x})}$$

$$= \lim_{h\to 0} \frac{-1}{\sqrt{1-x-h}+\sqrt{1-x}} = \boxed{\frac{-1}{2\sqrt{1-x}}}$$

at $x_0 = -3$, $\frac{dy}{dx} = \frac{-1}{2\sqrt{1+3}} = -1/4$

18. $f(x) = x^2$

a) If $x=-2$, then $y=4$. If $x=-1.9$, then $y=3.61$

Slope of secant line is $m_{sec} = \frac{4-3.61}{-2+1.9} = \frac{.39}{-.1} = -3.9$

b) $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^2 - x^2}{h} = \lim_{h\to 0} \frac{x^2+2xh+h^2-x^2}{h}$

$$= \lim_{h\to 0} \frac{2xh+h^2}{h} = \lim_{h\to 0} (2x+h) = 2x$$

Slope of tangent at $x=-2$ is $m = f'(-2) = -4$. The slope of the secant approximates the slope of the tangent pretty closely.

21. $P(x) = 400(15-x)(x-2)$.

a) $P'(x) = \lim_{h\to 0} \frac{P(x+h)-P(x)}{h} = \lim_{h\to 0} \frac{400(15-x-h)(x+h-2) - 400(15-x)(x-2)}{h}$

$$= \lim_{h\to 0} \frac{400(15x+15h-30-x^2-xh+2x-hx-h^2+2h) - 400(15x-30-x^2+2x)}{h}$$

$$= \lim_{h\to 0} \frac{400(17x-x^2-2xh+17h-h^2-30) - 400(-x^2+17x-30)}{h}$$

$$= \lim_{h\to 0} \frac{-800xh+6800h-400h^2}{h} = \lim_{h\to 0}(-800x+6800-400h)$$

$$= -800x+6800$$

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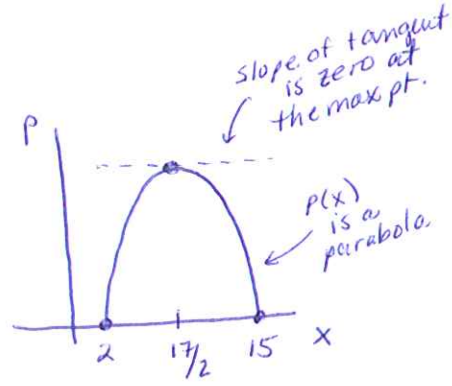
21. (cont.)

b) $P'(x)=0$ when $-800x+6800=0$

$$x = \frac{-6800}{-800} = 17/2$$

The profit $P(x)$ when $x = 17/2$ will be the maximum profit.



24. Profit = Revenue − Cost

Profit = (price per unit)(#units sold) − (cost per unit)(#units produced)

$$P(x) = x(120-x) - 20(120-x)$$
$$= 120x - x^2 - 2400 + 20x$$
$$= -x^2 + 140x - 2400$$ (notice graph is a parabola).

As in Problem #21, profit will be greatest when $P'(x)=0$. (horizontal tangent)

$$P'(x) = \lim_{h\to 0} \frac{P(x+h)-P(x)}{h} = \lim_{h\to 0} \frac{[-(x+h)^2+140(x+h)-2400]-[-x^2+140x-2400]}{h}$$

$$= \lim_{h\to 0} \frac{-x^2-2xh-h^2+140x+140h-2400+x^2-140x+2400}{h}$$

$$= \lim_{h\to 0} \frac{-2xh-h^2+140h}{h} = \lim_{h\to 0} (-2x-h+140)$$

$$= -2x+140$$

$$P'(x) = 0 = -2x+140$$
$$2x = 140$$
$$x = 70$$ Profit is greatest when $x = 70$ = price

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27. a) $f(x) = 3x-2$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{3(x+h)-2-(3x-2)}{h}$$
$$= \lim_{h\to 0} \frac{3x+3h-2-3x+2}{h} = \lim_{h\to 0} \frac{3h}{h} = 3$$

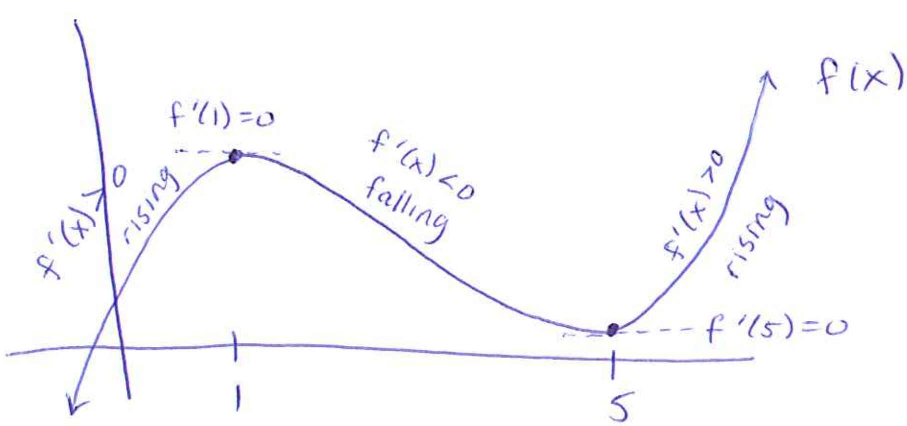
b) If $x=-1$, then $y=f(-1)=3(-1)-2)=-5$. Point $(-1,-5)$.
$m = f'(-1) = 3$. So tangent line is $y+5=3(x+1)$, or $y=3x-2$

c) We know the graph of $f(x)$ is a line. Any line tangent to this must be the original line.

31. The graph of $f(x)$ rises over $a \le x \le b$ if $f'(x)$ is positive there, because since slope is positive over the interval, the graph will go up. Positive slope means the function rises.

32. Sketch $f(x)$ so that:

$f'(x)>0$ when $x<1$ and when $x>5$ (rising)
$f'(x)<0$ when $1<x<5$ (falling)
$f'(1)=0$ and $f'(5)=0$ (horizontal tangent lines there).



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2. $y = x^{7/3}$ $\quad y' = \frac{7}{3}x^{4/3}$

6. $y = 3x^5 - 4x^3 + 9x - 6$ $\quad y' = 15x^4 - 12x^2 + 9$

12. $f(t) = 2\sqrt{t^3} + \frac{4}{\sqrt{t}} - \sqrt{2}$
$= 2t^{3/2} + 4t^{-1/2} - \sqrt{2}$
$f'(t) = 3t^{1/2} - 2t^{-3/2}$

19. $y = \sqrt{x^3} - x^2 + \frac{16}{x^2}$. Find tangent line at $(4, -7)$
$y = x^{3/2} - x^2 + 16x^{-2}$
$y' = \frac{3}{2}x^{1/2} - 2x - 32x^{-3}$
$m = y'(4) = \frac{3}{2}\sqrt{4} - 2(4) - \frac{32}{4^3}$
$= 3 - 8 - \frac{32}{64} = \frac{-10}{2} - \frac{1}{2} = \frac{-11}{2}$.
Line: $y + 7 = \frac{-11}{2}(x-4)$ $\quad$ or $\quad y = \frac{-11}{2}x + 15$

24. $f(x) = x(\sqrt{x} - 1)$. Find tangent line at $x = 4$
$f(x) = x^{3/2} - x$
$f'(x) = \frac{3}{2}x^{1/2} - 1$
$m = f'(4) = \frac{3}{2}(2) - 1 = 2$

Point: $(4, 4)$ $\quad y = 4(\sqrt{4} - 1) = 4 \cdot 1$
Line: $y - 4 = 2(x-4)$
or $y = 2x - 4$

28. $f(x) = \frac{x + \sqrt{x}}{x\sqrt{x}}$. Find $\frac{dy}{dx}$ at $x = 1$.
$f(x) = (x + x^{1/2})(x^{-3/2}) = x^{-1/2} + x^{-1}$.
$f'(x) = \frac{-1}{2}x^{-3/2} - x^{-2}$
$f'(1) = -\frac{1}{2} - 1 = -3/2 = \left.\frac{dy}{dx}\right|_{x=1}$

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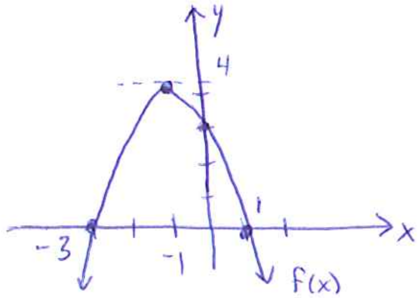
30. Sketch $f(x) = 3 - 2x - x^2$ and use calculus to find its highest point.



Highest point is where $f'(x) = 0$.

$f'(x) = -2 - 2x = 0$

$-2x = 2$

$x = -1$

$f(-1) = 3 + 8 - 1 = 4$. Highest point is $(-1, 4)$

44. Earnings $= A(t) = 0.1t^2 + 10t + 20$, where $t = $ # years past 1998.

a) $A'(t) = 0.2t + 10$. $A'(4) = 0.8 + 10 = $ 10.8 thousand dollars per year. (In 2002, notice $t = 4$)

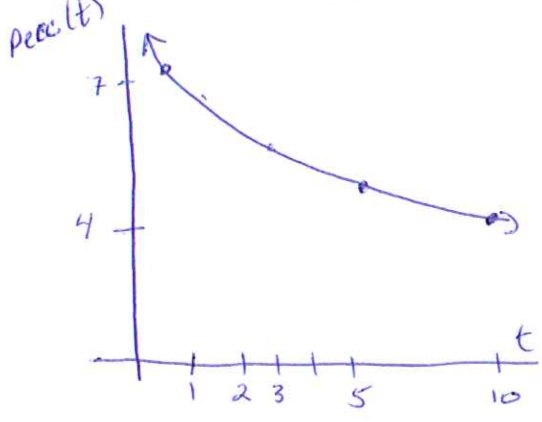
b) Percentage rate of growth $= \dfrac{100\,A'(t)}{A(t)}$. For $t = 4$, % rate of growth $= \dfrac{100\,(10.8)}{0.1(4)^2 + 10(4) + 20} = \dfrac{1080}{61.6} \approx$ 17.53% growth rate

47. Starting salary \$24,000, \$2000 raise each year.

a) Salary $= 24000 + 2000t$, where $t = $ # years passed.

$S' = 2000$.

% rate of growth $= \dfrac{100\,(2000)}{24000 + 2000t} = \dfrac{200000}{24000 + 2000t} = Perc(t)$



t	$Perc(t)$
1	$\frac{200000}{24000+2000} = 7.692\%$ ← b
2	$\frac{200000}{28000} = 7.14\%$
3	$\frac{200000}{30000} = 6.67\%$
5	$\frac{200000}{34000} = 5.88\%$
10	$\frac{200000}{44000} = 4.55\%$

c) Percentage rate of change gets smaller over time.