NAME KEY

Math 1212
Test 1
Fall 2015

You have 50 minutes to complete this test. You must *show all work* to receive full credit. Work any 7 of the following 8 problems. Clearly **CROSS OUT** the problem you do not wish me to grade. Each problem is worth 14 points, and you get 2 points for free, for a total of 100 points. If you have any questions, please come to the front and ask.

1. Using the definition of the derivative, find $f'(x)$ if $f(x)=x^3-4$.

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{\left((x+h)^3-4\right)-(x^3-4)}{h}$$

$$=\lim_{h\to 0}\frac{x^3+3x^2h+3xh^2+h^3-4-x^3+4}{h}$$

$$=\lim_{h\to 0}\frac{3x^2h+3xh^2+h^3}{h}=\lim_{h\to 0}\left(3x^2+3xh+h^2\right)$$

$$=3x^2$$

2. Evaluate the following limits. If any of them do not exist, EXPLAIN why not ("because it's undefined" and "denominator is zero" are not sufficient explanations).

(a) $\lim\limits_{x\to 3}\dfrac{x^2-2x-3}{9-x^2}=\lim\limits_{x\to 3}\dfrac{(x-3)(x+1)}{(3-x)(3+x)}=\lim\limits_{x\to 3}\dfrac{x+1}{-(3+x)}=\dfrac{4}{-6}=\dfrac{-2}{3}$

fill in, get $\frac{0}{0}$, not working!

(b) $\lim\limits_{x\to 2}\dfrac{4}{(x-2)^2}=\infty$

fill in, get $\frac{4}{0}$
not working
use chart

x	y
1	4/1 = 4
1.5	4/.4 = 16
1.9	4/.01 = 400
1.99	4/.0001 = 40000
3	4/1 = 4
2.5	4/.4 = 16
2.1	4/.01 = 400
2.01	4/.001 = 40000

$\lim\limits_{x\to 2^-}\dfrac{4}{(x-2)^2}=\infty$

same

$\lim\limits_{x\to 2^+}\dfrac{4}{(x-2)^2}=\infty$

(c) $\lim\limits_{x\to -1}\dfrac{2x}{x+5}=\dfrac{-2}{-1+5}=\dfrac{-2}{4}=\dfrac{-1}{2}$

3. Suppose that the total cost of producing x units of a product is given by $C(x)=\frac{1}{8}x^2+3x+98$, and that all x units will be sold if the price is set at $p(x)=25-\frac{1}{3}x$ dollars per unit.

a) Find an equation for revenue.
b) Find an equation for profit.
c) Using marginal analysis, estimate the profit obtained by the production and sale of the 6th unit.
d) Find the actual profit obtained by the production and sale of the 6th unit.

a) $R = p \cdot q = (25-\frac{1}{3}x)(x)$

$\boxed{R(x) = 25x - \frac{1}{3}x^2}$

b) $\boxed{P = R - C = 25x - \frac{1}{3}x^2 - (\frac{1}{8}x^2 + 3x + 98)}$

c) $P' = 25 - \frac{2}{3}x - \frac{1}{4}x - 3$

$P'(5) = 25 - \frac{10}{3} - \frac{5}{4} - 3 = 22 - \frac{40}{12} - \frac{15}{12} = \frac{264-40-15}{12} = \frac{209}{12}$

profit from 6th unit $\approx$ \$17.42

d) $P(6) - P(5) = \left[25(6) - \frac{1}{3}(36) - (\frac{1}{8}(36) + 18 + 98)\right]$

$- \left[25(5) - \frac{1}{3}(25) - (\frac{1}{8}(25) + 15 + 98)\right]$

$= \frac{407}{24} \approx$ \$16.96

4. Find $f'(x)$ (do not simplify!) if :

a) $f(x)=\left(2x^3-\frac{4}{x^2}+1\right)\left(\sqrt[3]{x}+5x-4\right) = (2x^3 - 4x^{-2} + 1)(x^{1/3} + 5x - 4)$

$f'(x) = (6x^2 + 8x^{-3})(x^{1/3} + 5x - 4) + (2x^3 - 4x^{-2} + 1)(\frac{1}{3}x^{-2/3} + 5)$

b) $f(x)=\frac{x-5x^6+4}{3x+2}$

$$f'(x) = \frac{(1-30x^5)(3x+2) - (x - 5x^6 + 4)(3)}{(3x+2)^2}$$

5. Suppose $f(x) = \begin{cases} Ax - 2 & x < 1 \\ x^2 - 4x + 4 & x \geq 1 \end{cases}$. Find the value for A that will make f continuous. Be sure to show your work and explain why your value makes f continuous.

$f(x)$ is a line when $x<1$, so f is continuous for all $x<1$.
$f(x)$ is a parabola when $x>1$, so f is continuous for all $x>1$.

we need the ends to match up.

$f(x)$ has a hole at $(1, A-2)$ fill $x=1$ into first part

$f(x)$ has a point at $(1,1)$ fill $x=1$ into 2nd part.

we need point = hole, so

$A-2=1$

$A=3$ will make f continuous for all x.

6. Find the equation of the line tangent to $f(x)=\sqrt[4]{x}-2x^2+5$ at the point where $x=1$.

$f(x) = x^{1/4} - 2x^2 + 5$

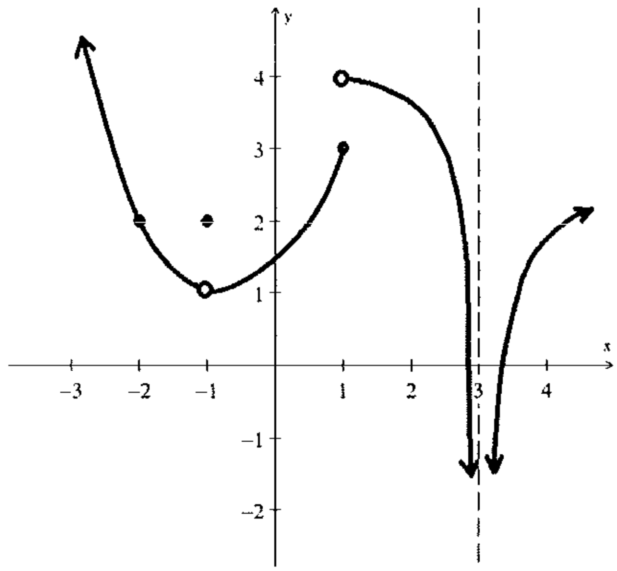
point: $x=1$
$y = 1-2+5 = 4$ $(1,4)$

slope: $m = f'(1)$
$f'(x) = \frac{1}{4}x^{-3/4} - 4x$
$m = f'(1) = \frac{1}{4} - 4 = \frac{-15}{4}$

line: $y-4 = \frac{-15}{4}(x-1)$ or $y = \frac{-15}{4}x + \frac{15}{4} + 4$

$y = \frac{-15}{4}x + \frac{31}{4}$

7. Consider the graph of the function $f(x)$ given below.



(a) For what values of x is $f(x)$ **not** continuous? $x = -1, 1, 3$

(b) Find $\lim_{x \to -2} f(x)$. $= 2$

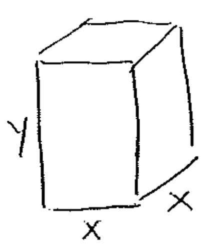
(c) Find $\lim_{x \to 1^-} f(x)$. $= 3$

(d) Find $\lim_{x \to 1^+} f(x)$. $= 4$

(e) Find $\lim_{x \to 1} f(x)$. DNE (not same)

(f) Find $\lim_{x \to 3} f(x)$. $= -\infty$

8. A rectangular box with no top and a **square** base is to be built for $48. The sides of the box will cost $3 per square meter, and the base with cost $4 per square meter. Express the volume of the box in terms of the length of the base.



Volume $= x^2 y$ get rid of this

$$\text{cost} = 48 = 3xy + 3xy + 3xy + 3xy + 4x^2$$
$$48 = 12xy + 4x^2$$
$$12 = 3xy + x^2$$
$$12 - x^2 = 3xy \longrightarrow y = \frac{12 - x^2}{3x}$$
$$\text{Volume} = x^2\left(\frac{12 - x^2}{3x}\right) = \frac{12x^2 - x^4}{3x}$$
$$V = 4x - \frac{1}{3}x^3$$