NAME KEY

Math 1212
Test 1
Fall 2016

You have 50 minutes to complete this test. You must *show all work* to receive full credit. Work any 7 of the following 8 problems. Clearly **CROSS OUT** the problem you do not wish me to grade. Each problem is worth 14 points, and you get 2 points for free, for a total of 100 points. If you have any questions, please come to the front and ask.

1. Using the definition of the derivative, find $f'(x)$ if $f(x)=\dfrac{8}{x+2}$.

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{\frac{8}{x+h+2}-\frac{8}{x+2}}{h}$$

$$=\lim_{h\to 0}\frac{(8x+16)-(8x+8h+16)}{(x+h+2)(x+2)}\cdot\frac{1}{h}$$

$$=\lim_{h\to 0}\frac{-8h}{(x+h+2)(x+2)}\cdot\frac{1}{h}=\lim_{h\to 0}\frac{-8}{(x+h+2)(x+2)}$$

$$=\frac{-8}{(x+2)(x+2)}=\frac{-8}{(x+2)^2}$$

2. Evaluate the following limits. If any of them do not exist, EXPLAIN why not ("because it's undefined" and "denominator is zero" are not sufficient explanations).

(a) $\displaystyle\lim_{x\to 6}\frac{x+1}{x^2-1}=\frac{6+1}{36-1}=\frac{7}{35}=\frac{1}{5}$

(b) $\displaystyle\lim_{x\to 4}\frac{\sqrt{x}-2}{x-4}\cdot\frac{\sqrt{x}+2}{\sqrt{x}+2}=\lim_{x\to 4}\frac{x-4}{(x-4)(\sqrt{x}+2)}=\lim_{x\to 4}\frac{1}{\sqrt{x}+2}=\frac{1}{4}$

(c) $\displaystyle\lim_{x\to 0}\frac{1}{x^2}=\pm\infty$

fill in, get $\frac{1}{0}$, so make a chart.

x	$f(x)$
1	1
0.1	$\frac{1}{.01}=100$
0.01	$\frac{1}{.0001}=10000$
	∞

x	$f(x)$
-1	1
-0.1	100
-0.01	10000
	∞

3. The cost function for flavored coffee at an upscale coffeehouse is given in dollars by $C(x) = 3x + 160$, where x is the number of pounds of coffee. If coffee can be sold for \$7 per pound, how many pounds will have to be sold in order to break even? What will be the revenue at this point?

To break even, Revenue = Cost (or profit = R − C = 0)

price·quant = cost

$7x = 3x + 160$

$4x = 160$

$x = 40$ pounds to break even

Revenue when $x = 40$ pounds is

$R(x) = 7x$

$R(40) = 7(40) = \$280$

4. Find $f'(x)$ if:

a) $f(x) = (x^2 - 4x + 2)(5x + \sqrt[3]{x}) = (x^2 - 4x + 2)(5x + x^{1/3})$

$f'(x) = (2x - 4)(5x + x^{1/3}) + (x^2 - 4x + 2)\left(5 + \frac{1}{3}x^{-2/3}\right)$

b) $f(x) = 4x^3 - 3\sqrt{x} - 15x^2 + \frac{7}{x} = 4x^3 - 3x^{1/2} - 15x^2 + 7x^{-1}$

$f'(x) = 12x^2 - \frac{3}{2}x^{-1/2} - 30x - 7x^{-2}$

5. Suppose that the price in dollars of a stereo system is given by $p(x) = \frac{1000}{x^2} + 1000$, where x is the demand (number of stereos sold). Find the marginal revenue when $x = 10$. Then write a sentence describing the meaning of this marginal revenue.

Revenue = price · quantity

$R(x) = p(x) \cdot x$

$R(x) = \left(\frac{1000}{x^2} + 1000\right)(x)$

$R(x) = 1000x^{-1} + 1000x$

$R'(x) = -1000x^{-2} + 1000$

$R'(10) = \frac{-1000}{100} + 1000$

$= \$990$

When 10 units have already been sold, the estimated revenue from the production and sale of the next (11th) unit is \$990.

6. Find the equation of the line tangent to $f(x) = \frac{(2x-5)(x+7)}{x^2+3}$ at the point where $x = 1$.

Point: $x = 1,\ y = \frac{(2-5)(1+7)}{1+3} = \frac{-3(8)}{4} = -6 \qquad (1, -6)$

Slope: $f'(x) = \frac{[(2)(x+7) + (2x-5)(1)](x^2+3) - (2x-5)(x+7)(2x)}{(x^2+3)^2}$

$m = f'(1) = \frac{[2(8) - 3](4) - (-3)(8)(2)}{16} = \frac{13(4) + 48}{16} = \frac{100}{16} = \frac{25}{4}$

Line: $y + 6 = \frac{25}{4}(x - 1)$ or $y = \frac{25}{4}x - \frac{25}{4} - \frac{24}{4}$

$y = \frac{25}{4}x - \frac{49}{4}$

7. Consider the graph of the function $f(x)$ given below.

(a) Find $\lim_{x\to 0} f(x)$. $=1$

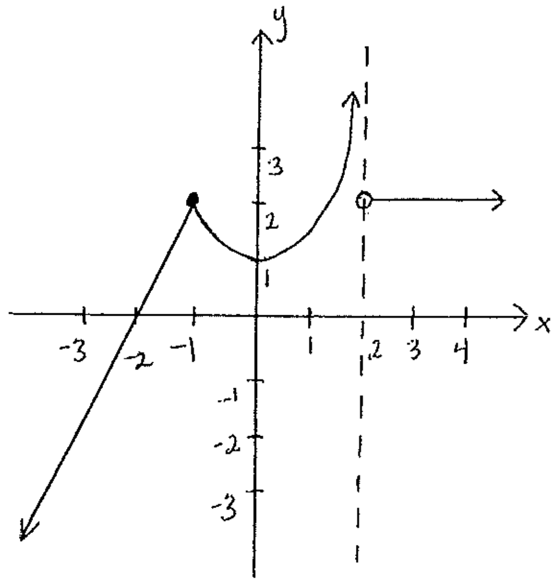
(b) Find $\lim_{x\to -1} f(x)$. $=2$

(c) Find $\lim_{x\to 2^-} f(x)$. $=\infty$

(d) Find $\lim_{x\to 2^+} f(x)$. $=2$

(e) Find $\lim_{x\to 2} f(x)$. DNE

(f) Find $\lim_{x\to 4} f(x)$. $=2$



8. Consider the function $f(x)=\begin{cases} x+1 & \text{if} \quad x<1 \\ x^2-3x+4 & \text{if} \quad 1\le x\le 3 \\ 5-x & \text{if} \quad x>3 \end{cases}$. Describe where the function is continuous and where it is discontinuous. Be sure to fully explain your answers.

For all x-values except $x=1$ and $x=3$, $f(x)$ is continuous, since each piece is a polynomial. So we only need to consider $x=1$ and $x=3$.

$\underline{x=1}$: $f(1)=1^2-3+4=2$ exists ✓

$\left.\begin{array}{l}\lim_{x\to 1^-} f(x)=\lim_{x\to 1^-}(x+1)=2 \\ \lim_{x\to 1^+} f(x)=\lim_{x\to 1^+}(x^2-3x+4)=1^2-3+4=2\end{array}\right\}$ ✓

Notice $\lim_{x\to 1} f(x)=f(1)$, $\boxed{\text{so } f \text{ is cont. at } x=1}$ ✓

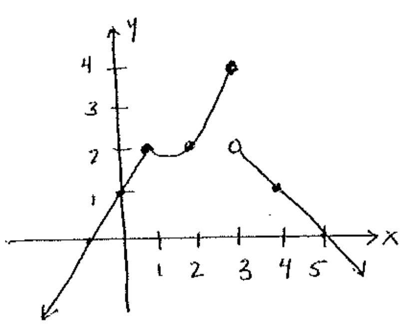
$\underline{x=3}$: $f(3)=(3)^2-3(3)+4=4$ exists ✓

$\lim_{x\to 3^-} f(x)=\lim_{x\to 3^-}(x^2-3x+4)=3^2-3x+4=4$

$\lim_{x\to 3^+} f(x)=\lim_{x\to 3^+}(5-x)=2$

different! limit DNE

So $\boxed{f \text{ is NOT continuous at } x=3}$



from graph, ok to say f is continuous for all $x\neq 3$.