NAME KEY

Math 1212
Test 1
Spring 2015

You have 50 minutes to complete this test. You must *show all work* to receive full credit. Work any 7 of the following 8 problems. Clearly **CROSS OUT** the problem you do not wish me to grade. Each problem is worth 14 points, and you get 2 points for free, for a total of 100 points. If you have any questions, please come to the front and ask.

1. Using the definition of the derivative, find $f'(x)$ if $f(x)=\dfrac{3}{2x-1}$.

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{\frac{3}{2(x+h)-1}-\frac{3}{2x-1}}{\frac{h}{1}}$$

$$=\lim_{h\to 0}\frac{3(2x-1)-3(2x+2h-1)}{(2x+h-1)(2x-1)}\cdot\frac{1}{h}$$

$$=\lim_{h\to 0}\frac{6x-3-6x-6h+3}{h(2x+h-1)(2x-1)}=\lim_{h\to 0}\frac{-6h}{h(2x+h-1)(2x-1)}=\frac{-6}{(2x-1)^2}$$

2. Evaluate the following limits. If any of them do not exist, EXPLAIN why not ("because it's undefined" and "denominator is zero" are not sufficient explanations).

(a) $\lim_{x\to 3}\dfrac{2x+1}{x^2-1}=\dfrac{2(3)+1}{9-1}=\dfrac{7}{8}$

(b) $\lim_{x\to 4}\dfrac{\sqrt{x}-2}{x-4}\cdot\dfrac{\sqrt{x}+2}{\sqrt{x}+2}=\lim_{x\to 4}\dfrac{x-4}{(x-4)(\sqrt{x}+2)}=\lim_{x\to 4}\dfrac{1}{\sqrt{x}+2}=\dfrac{1}{4}$

$\frac{0}{0}$, need to cancel something...

(c) $\lim_{x\to 1}\dfrac{x+1}{(x-1)^2}=\infty$

$\frac{2}{0}$, use a chart...

x	y	
0	1	if $x\to 1^-$, $y\to\infty$
$\frac{1}{2}$	$\frac{3}{2}/\frac{1}{4}=6$	
.9	$1.9/\frac{1}{100}=190$	
.99	$1.99/\frac{1}{10000}=19900$	
2	$\frac{3}{1}=3$	if $x\to 1^+$, $y\to\infty$
1.5	$2.5/\frac{1}{4}=10$	
1.1	$2.1/\frac{1}{100}=210$	
1.01	$2.01/\frac{1}{10000}=20100$	

3. During the summer, a group of students builds kayaks in a converted garage. The rental for the garage is \$1500 for the whole summer, and the cost for materials to build one kayak is \$125. Each kayak can be sold for \$275. How many kayaks must the students sell in order to break even?

"Break even" means money in = money out, or revenue = cost

Let x = # kayaks.

$275x = 1500 + 125x$

$150x = 1500$

$x = 10$ kayaks to break even

4. Find, but DO NOT simplify, $f'(x)$ if:

a) $f(x) = (x^2+2)(x+\sqrt{x}) = (x^2+2)(x+x^{1/2})$

$f'(x) = 2x(x+x^{1/2}) + (x^2+2)(1+\frac{1}{2}x^{-1/2})$

b) $f(x) = \frac{4x^3-3\sqrt{x}}{10x+2} - 15x^2+7 = \frac{4x^3-3x^{1/2}}{10x+2} - 15x^2+7$

$f'(x) = \frac{(12x^2-\frac{3}{2}x^{-1/2})(10x+2) - (4x^3-3x^{1/2})(10)}{(10x+2)^2} - 30x$

5. Suppose that the total cost of producing x units of a product is given by $C(x)=\frac{1}{8}x^2+3x+98$, and that all x units will be sold if the price is set at $p(x)=25-\frac{1}{3}x$ dollars per unit.

a) Find an equation for revenue.

$R = p \cdot q$. $\quad R(x) = \left(25-\frac{1}{3}x\right)(x) = 25x-\frac{1}{3}x^2$

b) Find an equation for profit.

$P = R - C$ $\quad P(x) = 25x-\frac{1}{3}x^2-\left(\frac{1}{8}x^2+3x+98\right)$

$= -\frac{11}{24}x^2+22x-98$

c) Using marginal analysis, estimate the profit obtained by the production and sale of the 6th unit.

$P'(x) = 25-\frac{2}{3}x-\frac{1}{4}x-3$

$P'(5) = 25-\frac{10}{3}-\frac{5}{4}-3 = 22-\frac{40}{12}-\frac{15}{12} = \frac{264-55}{12}$

$= \$\frac{209}{12} \approx 17.42$

d) Find the actual profit obtained by the production and sale of the 6th unit.

$P(6)-P(5) = \left(-\frac{11}{24}(36)+22(6)-98\right)-\left(-\frac{11}{24}(25)+22(5)-98\right)$

$= 17.5-\left(-\frac{275}{24}+110-98\right) = 17.5-(+0.54)$

$\approx \$16.96$

6. Find the equation of the line tangent to $f(x)=\frac{(2x-5)(x+7)}{x^2+3}$ at the point where $x=1$.

Point: $x=1$, $y=\frac{(2-5)(1+7)}{1+3}=\frac{-3\cdot 8}{4}=-6$ $\quad (1,-6)$

Slope: $f'(x)=\frac{[2(x+7)+(2x-5)(1)](x^2+3)-(2x-5)(x+7)(2x)}{(x^2+3)^2}$

$m=f'(1)=\frac{(2(8)+(-3))(4)-(-3)(8)(2)}{16}=\frac{13\cdot 4+48}{16}$

$=\frac{25}{4}$

Line: $y+6=\frac{25}{4}(x-1)$ $\quad \left(\text{or } y=\frac{25}{4}x-\frac{49}{4}\right)$

7. Consider the graph of the function $f(x)$ given below.

a) For what values of x is $f(x)$ discontinuous? $x=0,\ x=2$

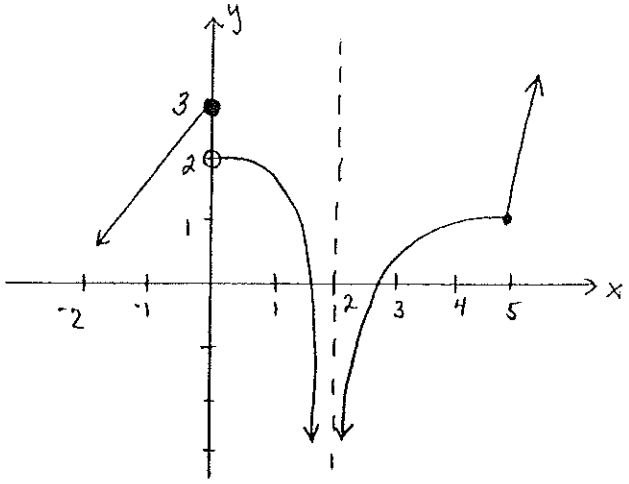
b) Find $\lim_{x\to 5} f(x)$. $= 1$

c) Find $\lim_{x\to 0^-} f(x)$. $= 3$

d) Find $\lim_{x\to 0^+} f(x)$. $= 2$

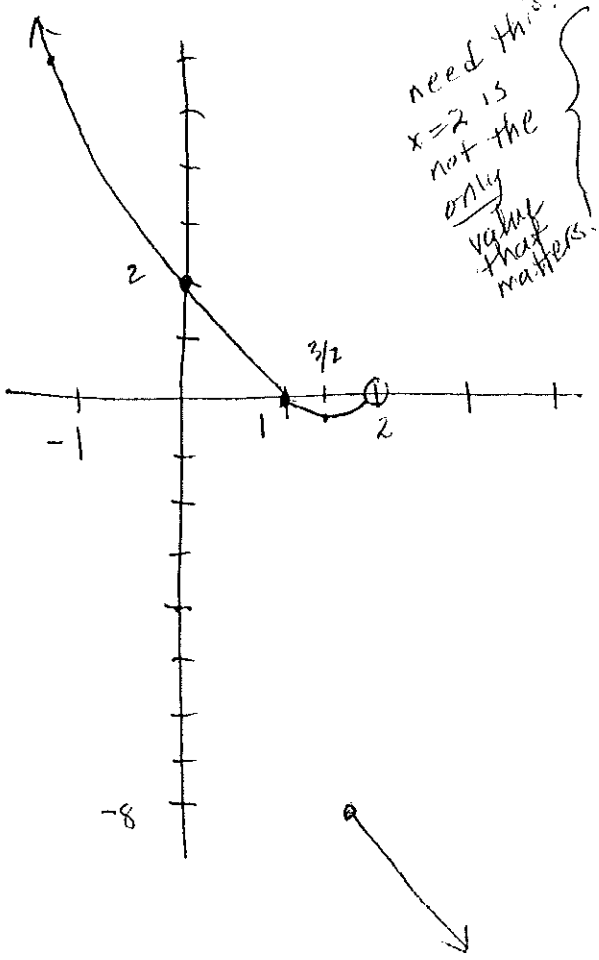
e) Find $\lim_{x\to 0} f(x)$. DNE

f) Find $\lim_{x\to 2} f(x)$. $= -\infty$



$(x-2)(x-1)$

8. Sketch a nice big graph of $f(x)=\begin{cases} x^2-3x+2 & \text{if } x<2 \\ -5x+2 & \text{if } x\geq 2 \end{cases}$. Be sure to clearly label points and axes. Fully describe the continuity of this function.



need this! $x=2$ is not the only value that matters.

$f(x)$ is continuous for all x except maybe $x=2$, since each piece is a polynomial.

Now consider $x=2$.

Notice

$\lim_{x\to 2^-} f(x) = 0$, and

$\lim_{x\to 2^+} f(x) = -8$. SO

$\lim_{x\to 2} f(x)$ DNE, and

$f(x)$ is discontinuous at $x=2$.

(or, notice graph is broken!)