NAME KEY

Math 1212
Test 1
Summer 2015

You have 60 minutes to complete this test. You must *show all work* to receive full credit. Work any 7 of the following 8 problems. Clearly **CROSS OUT** the problem you do not wish me to grade. Each problem is worth 14 points, and you get 2 points for free, for a total of 100 points. If you have any questions, please come to the front and ask.

1. Using the definition of the derivative, find $f'(x)$ if $f(x) = 2 - \frac{x}{4} - x^2$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\left(2 - \frac{x+h}{4} - (x+h)^2\right) - \left(2 - \frac{x}{4} - x^2\right)}{h}$$

$$= \lim_{h\to 0} \frac{\frac{-x-h}{4} - x^2 - 2xh - h^2 + \frac{x}{4} + x^2}{h} = \lim_{h\to 0} \frac{-\frac{h}{4} - 2xh - h^2}{h}$$

$$= \lim_{h\to 0} \left(-\frac{1}{4} - 2x - h\right) = \boxed{-\frac{1}{4} - 2x}$$

2. Evaluate the following limits. If any of them do not exist, EXPLAIN why not ("because it's undefined" and "denominator is zero" are not sufficient explanations).

(a) $\lim_{x\to 3^-} \frac{x+3}{x^2-9} = \lim_{x\to 3^-} \frac{x+3}{(x+3)(x-3)} = \lim_{x\to 3^-} \frac{1}{x-3}$ fill in, get $\frac{1}{0}$, need chart

fill in, get $\frac{6}{0}$, need chart $= \boxed{-\infty}$

x	$f(x)$
2	-1
2.5	-2
2.9	-10
2.99	-100 $\to -\infty$

(b) $\lim_{x\to -5} \frac{2x^2+9x-5}{x^2+5x} = \lim_{x\to -5} \frac{(2x-1)(x+5)}{x(x+5)}$

$= \lim_{x\to -5} \frac{2x-1}{x} = \frac{-10-1}{-5} = \boxed{\frac{11}{5}}$

(c) $\lim_{x\to 1} \frac{2x^2+x-3}{x^3+4} = \frac{2+1-3}{1+4} = \frac{0}{5} = \boxed{0}$

3. Find the equation of the line through (4,-7) and perpendicular to the line $3x+2y=1$.

$$3x+2y=1$$
$$2y=-3x+1$$
$$y=\frac{-3}{2}x+\frac{1}{2}$$

old $m=-3/2$, so perpendicular $m=2/3$, point is $(4,-7)$.

Line: $y+7=\frac{2}{3}(x-4)$ ← this is fine.

$$y=\frac{2}{3}x-\frac{8}{3}-7$$
$$y=\frac{2}{3}x-\frac{29}{3}$$

4. Find y' for the following functions (do not simplify) :

a) $y=\left(8x^2-3\sqrt{x}+\frac{3}{4x^2}\right)(5x^{-3}+7) = (8x^2-3x^{1/2}+\frac{3}{4}x^{-2})(5x^{-3}+7)$

$$y'=(16x-\frac{3}{2}x^{-1/2}-\frac{3}{2}x^{-3})(5x^{-3}+7) + (8x^2-3x^{1/2}+\frac{3}{4}x^{-2})(-15x^{-4})$$

b) $y=\frac{\sqrt[3]{x}+1}{3x^4-5} = \frac{x^{1/3}+1}{3x^4-5}$

$$y'=\frac{(\frac{1}{3}x^{-2/3})(3x^4-5)-(x^{1/3}+1)(12x^3)}{(3x^4-5)^2}$$

5. Suppose a company produces x custom tablets each week, and it costs the company \$350 per tablet to produce them. Suppose the company sells each tablet for $800 - x$ dollars, and at that price all of the tablets will be sold.

a) Find the revenue equation. rev = price · quantity

$R(x) = (800-x)(x) = 800x - x^2$

b) Find the profit equation. profit = Revenue − Cost

$P(x) = (800x - x^2) - 350x = 450x - x^2$

c) What is marginal profit?

$P'(x) = 450 - 2x$

d) If the company is currently producing 160 netbooks per week, should it increase or decrease production in order to raise its profit? Explain your answer.

$P'(x) = 450 - 2x$

$P'(160) = 450 - 320 = \$130$

Production should be increased, since the next tablet will bring in \$130 extra profit. (keep making more until marginal profit is zero.)

6. Find the equation of the line tangent to $f(x) = \dfrac{\sqrt{x}(2-x^2)}{x}$ at the point where $x = 4$.

point: $x = 4$

$y = f(4) = \dfrac{\sqrt{4}(2-16)}{4} = \dfrac{2 \cdot -14}{4} = -7$

$(4, -7)$

slope: $f(x) = x^{1/2}(2-x^2)(x^{-1}) = x^{1/2}(2x^{-1} - x) = 2x^{-1/2} - x^{3/2}$

$f'(x) = -x^{-3/2} - \frac{3}{2}x^{1/2}$

$f'(4) = -4^{-3/2} - \frac{3}{2}\sqrt{4} = -\dfrac{1}{(\sqrt{4})^3} - 3 = -\dfrac{1}{2^3} - 3$

$= -\frac{1}{8} - 3 = \frac{-25}{8} = m$

line: $y + 7 = \frac{-25}{8}(x-4)$

7. Consider the graph of the function $f(x)$ given below.

a) Find $\lim_{x\to 1} f(x)$. 2

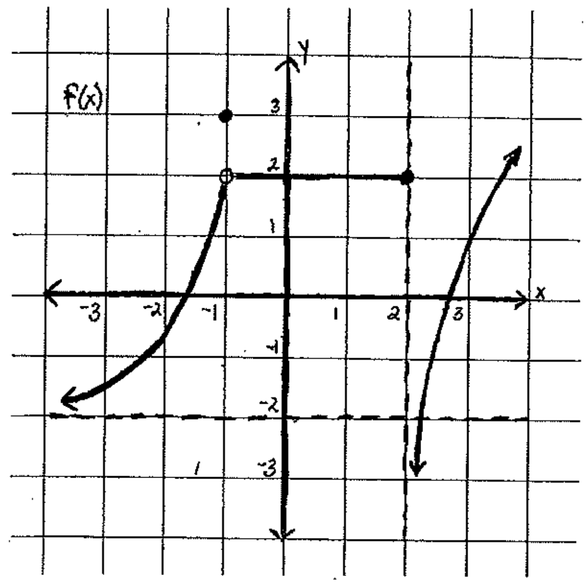
b) Find $\lim_{x\to 2^+} f(x)$. $-\infty$

c) Find $\lim_{x\to 2^-} f(x)$. 2

d) Find $\lim_{x\to 2} f(x)$. DNE

e) Find $\lim_{x\to -1} f(x)$. 2

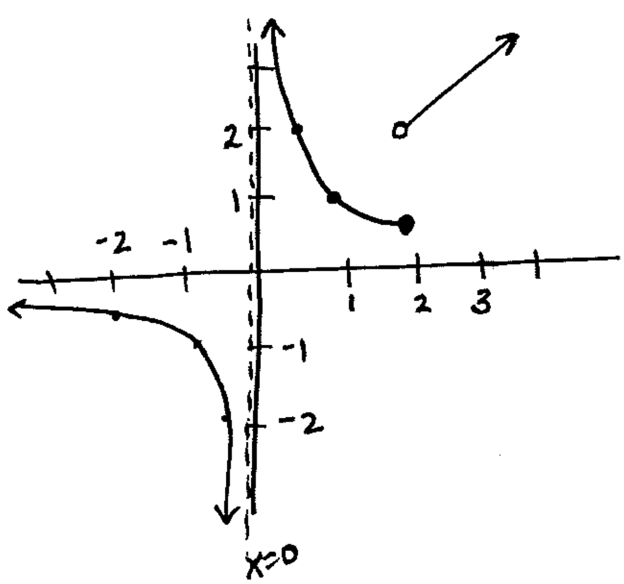
f) Find $\lim_{x\to -\infty} f(x)$. -2



g) List the intervals where $f(x)$ is continuous.

f (x) is continuous on $(-\infty,-1) \cup (-1,2) \cup (2,\infty)$

8. Graph the function $f(x)=\begin{cases} \frac{1}{x} & \text{if } x \le 2 \\ x & \text{if } x > 2 \end{cases}$. Be sure your graph is large enough for me to see and that it is clearly labeled. Then describe the continuity of the function based on your graph.



f is continuous on
$(-\infty,0) \cup (0,2) \cup (2,\infty)$