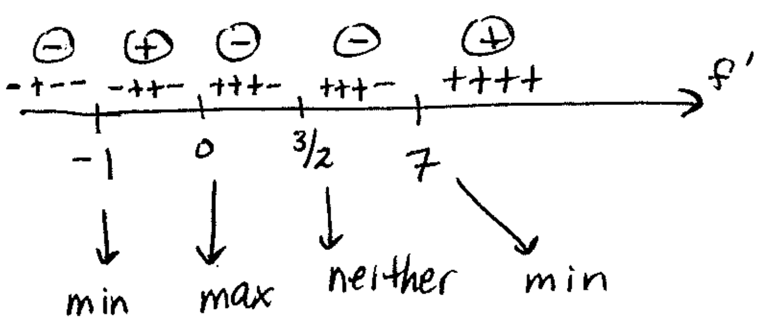
NAME <u>KEY</u>

Math 12
Test 2
Fall 2015

You have 50 minutes to complete this test. You must *show all work* to receive full credit. Work any 7 of the following 8 problems. Clearly **CROSS OUT** the problem you do not wish me to grade. Each problem is worth 14 points, and you get 2 points for free, for a total of 100 points. The answers will be posted on the electronic reserves later today.

1. Suppose that for a given function $f(x)$, the derivative has **already been calculated** to be $f'(x)=x^3(2x-3)^2(x+1)^5(x-7)$. Using this information, find all critical numbers of the original function f, list the intervals of increase and decrease, and tell whether each critical number will result in a maximum, a minimum, or neither.

critical numbers: $x=0,\ 3/2,\ -1,\ 7$



f is increasing on $(-1,0)\cup(7,\infty)$

f is decreasing on $(-\infty,-1)\cup(0,3/2)\cup(3/2,7)$

2. For the following functions, find all horizontal and vertical asymptotes (remember that an asymptote is a LINE, not a number). If there are no asymptotes, say so.

(a) $f(x)=\dfrac{x+2}{x^2}$

vertical: $x^2=0 \longrightarrow \boxed{x=0}$

horizontal: $\lim\limits_{x\to\infty}\dfrac{x}{x^2}=0 \longrightarrow \boxed{y=0}$

(b) $f(x)=\dfrac{x^2}{x+2}$

vertical: $x+2=0 \longrightarrow \boxed{x=-2}$

horizontal: $\lim\limits_{x\to\infty}\dfrac{x^2}{x}=\infty,\ \lim\limits_{x\to-\infty}\dfrac{x^2}{x}=-\infty$ $\boxed{\text{NO horiz}}$

(c) $f(x)=\dfrac{2x^2-9}{x^2+1}$

vertical: $x^2+1=0 \longrightarrow \boxed{\text{No vertical}}$

horizontal: $\lim\limits_{x\to\infty}\dfrac{2x^2}{x^2}=2 \longrightarrow \boxed{y=2}$

3. Suppose that the price of a product increases by 1%.

a) If the demand for the produce decreases by MORE than 1%, the demand for the product is called elastic. An example of a product like this is luxury cars.

b) If the demand for the product decreases by LESS than 1%, the demand for the product is called inelastic. An example of a product like this is milk.

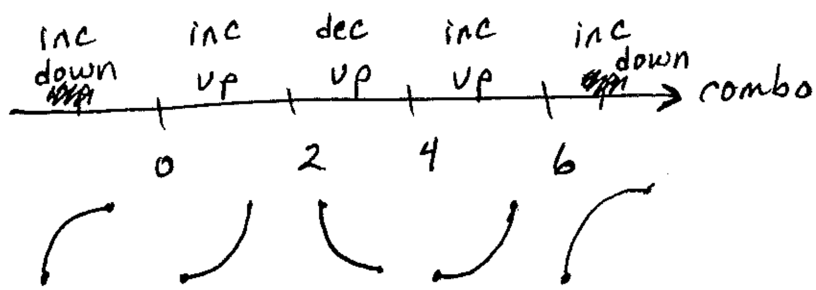
4. Sketch a nice BIG graph of a function with all the properties listed below. Make sure your graph is clearly labeled.

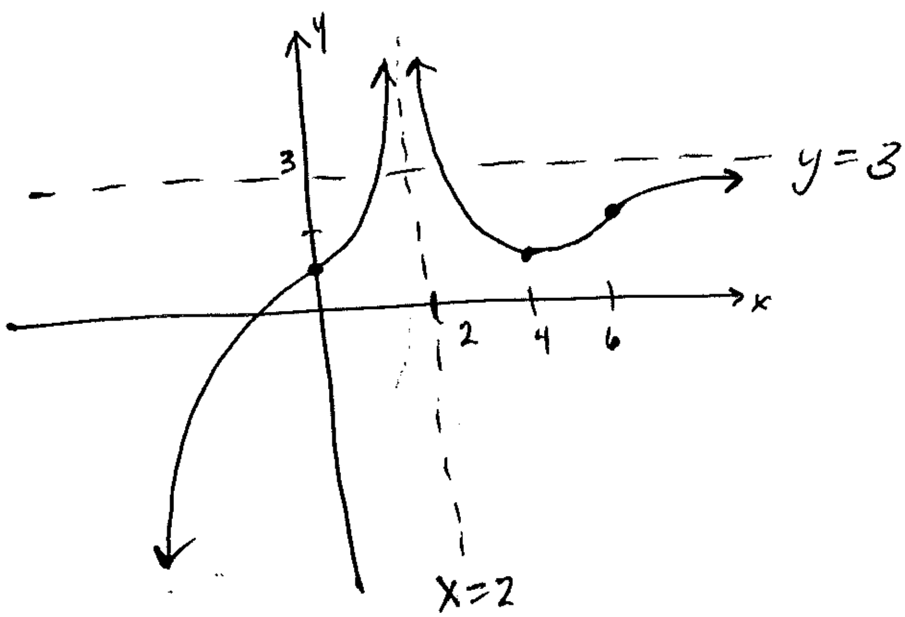
a) $f'(x) < 0$ for $2 < x < 4$, but $f'(x) \geq 0$ otherwise

b) $f''(x) < 0$ for $x < 0$ and also for $x > 6$, but $f''(x) \geq 0$ otherwise

c) $f(x)$ is undefined when $x = 2$ ← VA or hole

d) $\lim_{x \to \infty} f(x) = 3$. ← HA at y=3, on right side





5. Find $f'(x)$ for the following functions. DO NOT simplify!

(a) $f(x)=\left(x+\frac{1}{x}\right)^2-\frac{5}{\sqrt{3x}} = (x+x^{-1})^2 - \frac{5}{\sqrt{3}}x^{-1/2}$

$$f'(x) = 2(x+x^{-1})(1-x^{-2}) + \frac{5}{2\sqrt{3}}x^{-3/2}$$

(b) $f(x)=\sqrt{\frac{1-2x}{3x^5+2}} = \left(\frac{1-2x}{3x^5+2}\right)^{1/2}$

$$f'(x) = \frac{1}{2}\left(\frac{1-2x}{3x^5+2}\right)^{-1/2}\left(\frac{(-2)(3x^5+2)-(1-2x)(15x^4)}{(3x^5+2)^2}\right)$$

6. Find the equation of the line tangent to $(3xy^2+1)^4=2x-3y$ at the point $\left(\frac{1}{2},0\right)$.

$$4(3xy^2+1)^3\left((3)(y^2)+(3x)(2yy')\right) = 2-3y'$$

$x=1/2$, $y=0$. Fill in to get y' = slope

$$4(1)^3(0+0) = 2-3y'$$

$$0 = 2-3y'$$

$$-2 = -3y'$$

$$y' = 2/3$$

$$y-0 = \frac{2}{3}(x-1/2) \quad \text{or} \quad y = \frac{2}{3}x-\frac{1}{3}$$

7. Suppose that the speed of traffic on Highway 63 near the Pine Street intersection has been measured and the data suggest that between 1:00 pm and 6:00 pm on a normal weekday, the traffic speed is given by $s(t)=t^3-10.5t^2+30t+20$ miles per hour, where t is the number of hours past noon. At what time between 1:00 pm and 6:00 pm is the traffic moving the fastest, and how fast is it moving at that time?

$1 \le t \le 6$ (endpoints)

$s'(t) = 3t^2 - 21t + 30$

$= 3(t^2 - 7t + 10)$

$= 3(t-2)(t-5)$

Crit #'s : $t = 2, 5$

$s(1) = 40.5$

$s(2) = 46$ ← max

$s(5) = 32.5$

$s(7) = 38$

The maximum speed of 46 mph is reached at 2:00 pm.

8. A bus company will charter a bus that holds at most 50 people to groups of 35 or more. If a group contains exactly 35 people, each person pays \$60. In larger groups, everyone's fare is reduced \$1 for each person in excess of 35. For which size group will the bus company's revenue be the greatest? What will this total revenue be? (Make sure your answer is PRACTICAL, and use calculus – if you "plug & chug" you might get the right answer, but for no credit).

Revenue = (# people)(fare)   Let x = # extra people

$= (35+x)(60-x)$

$= 2100 + 25x - x^2$

$R' = 25 - 2x = 0$

CN : $x = 12.5$

$R'' = -2 < 0$, concave down, so max.

But 12.5 people makes no sense!

$R(0) = 2100$

$R(12) = 2256$

$R(13) = 2256$

$R(15) = 2250$

Total # people is either 47 or 48, and the maximum revenue is \$2256