NAME KEY

Math 1212
Test 2
Spring 2015

You have 50 minutes to complete this test. You must *show all work* to receive full credit. Work any 7 of the following 8 problems. Clearly **CROSS OUT** the problem you do not wish me to grade. Each problem is worth 14 points, and you get 2 points for free, for a total of 100 points. The answers will be posted on the electronic reserves later today.

1. Suppose $f(x)=\dfrac{x^2}{x-2}$. Find all critical numbers, list the intervals of increase and decrease, and tell whether each critical number will result in a maximum, a minimum, or neither. You do not need to find the y-values for the extrema.

$$f'(x)=\frac{(2x)(x-2)-x^2}{(x-2)^2}=\frac{x^2-4x}{(x-2)^2}=\frac{x(x-4)}{(x-2)^2}$$

CN: $x=0,2,4$

f': $+$ on $(-\infty,0)$, $-$ on $(0,2)$, $-$ on $(2,4)$, $+$ on $(4,\infty)$

0 2 4

increasing on $(-\infty,0)\cup(4,\infty)$
decreasing on $(0,2)\cup(2,4)$

$x=0$ gives a max
$x=2$ gives neither (flat spot)
$x=4$ gives a min

2. For the following functions, find all horizontal and vertical asymptotes (remember that an asymptote is a LINE, not a number). If there are no asymptotes, say so.

(a) $f(x)=\dfrac{2x^2+3x+1}{3x^2-5x+2}=\dfrac{(2x+1)(x+1)}{(3x-2)(x-1)}$ $\quad x=2/3$

VA: $x=2/3$
$x=1$

HA: $y=2/3$

(b) $f(x)=\dfrac{x+2}{x^2-4}=\dfrac{x+2}{(x+2)(x+2)}$ $\quad x=2$ (hole)

VA: $x=2$
HA: $y=0$

(c) $f(x)=x-\dfrac{1}{x}$

$$=\frac{x^2-1}{x}=\frac{(x+1)(x-1)}{x}$$

VA: $x=0$
HA: none

3. Suppose that $q(p) = 200 - 2p^2$ units of a product are demanded when the price is set at p dollars per unit, assuming $0 \le p \le 250$.

a) Calculate the elasticity of demand when $p = 6$.

$$E(p) = \frac{p}{q} \cdot q' = \frac{p}{200-2p^2} \cdot (-4p) = \frac{-4p^2}{200-2p^2} = \frac{-2p^2}{100-p^2}$$

$$E(6) = \frac{-72}{100-36} = \frac{-72}{64} = \frac{-9}{8}$$

b) Is the demand for the product elastic or inelastic at $p = 6$?

$|E(6)| = |{-9/8}| = 9/8 > 1$, so demand is elastic

c) Give an example of a product in the correct price range whose demand function would, in general, behave as in (a).

costs about \$6, luxury ... (many choices)

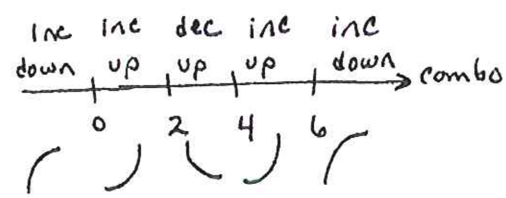
4. Sketch a nice BIG graph of a function with all the properties listed below. Make sure your graph is clearly labeled.

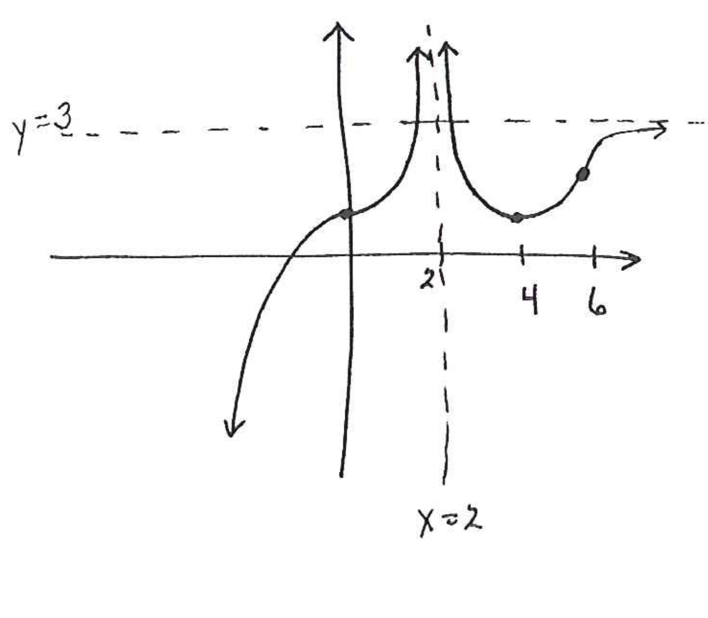
a) $f'(x) < 0$ for $2 < x < 4$, but $f'(x) \ge 0$ otherwise

b) $f''(x) < 0$ for $x < 0$ and also for $x > 6$, but $f''(x) \ge 0$ otherwise

c) $f(x)$ is undefined when $x = 2$ VA or hole

d) $\lim_{x \to \infty} f(x) = 3$. HA $y = 3$ (on right side)





5. Find $f'(x)$ for the following functions. DO NOT simplify!

(a) $f(x) = x^2(3-2x)^3$

$$f'(x) = 2x(3-2x)^3 + x^2(3)(3-2x)^2(-2)$$

(b) $f(x) = \sqrt{\dfrac{1-2x}{3x-2}} = \left(\dfrac{1-2x}{3x-2}\right)^{1/2}$

$$f'(x) = \frac{1}{2}\left(\frac{1-2x}{3x-2}\right)^{-1/2}\left(\frac{(-2)(3x-2)-(1-2x)(3)}{(3x-2)^2}\right)$$

6. Find the equation of the line tangent to the curve $(3xy^2+1)^4 = 2x-3y$ at the point $\left(\frac{1}{2},0\right)$.

$$4(3xy^2+1)^3(3y^2+6xyy') = 2-3y'$$

Fill in $x = 1/2$, $y = 0$

$$4(0+1)^3(0+0) = 2-3y'$$

$$0 = 2-3y'$$

$$3y' = 2$$

$$y' = 2/3 = m$$

Line: $y = \frac{2}{3}\left(x-\frac{1}{2}\right)$

7. Find the absolute minimum and absolute maximum *points* of $f(x)=\frac{1}{3}x^3-9x+2$ on the interval $0 \le x \le 2$.

$f'(x) = x^2-9 = (x+3)(x-3)$

CN : $x = 3, -3$. Notice these are both outside the interval.

So we only need to check the endpoints.

$f(0) = 2$

$f(2) = \frac{8}{3} - 18 + 2 = \frac{8}{3} - \frac{48}{3} = \frac{-40}{3}$

absolute min $(2, {-40}/3)$

absolute max $(0, 2)$

8. Mrs. Jones runs a small insurance company that sells policies for a large firm. Mrs. Jones does not sell policies herself, but she is paid a commission of \$50 for each policy sold by her employees. When she employs m salespeople, her company will sell q policies each week, where $q = m^3 - 12m^2 + 60m$. She pays her employees \$750 per week, and her weekly fixed costs are \$2500. Her office can accommodate at most 7 employees. How many employees should she have in order to maximize her weekly profit?

Profit = Revenue − cost

P = 50 (# policies) − 750 (# employees) − 2500

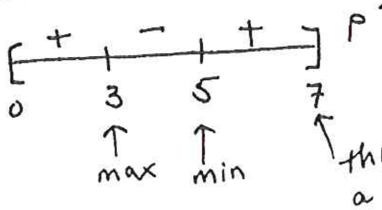
$P = 50(m^3 - 12m^2 + 60m) - 750m = 2500$

$P = 50m^3 - 600m^2 + 2250m - 2500$

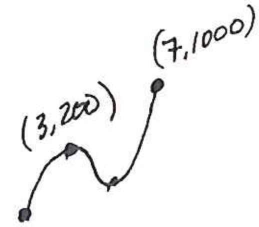
$P' = 150m^2 - 1200m + 2250$

$P' = 150(m^2 - 8m + 15) = 150(m-5)(m-3)$

CN : $m = 3, 5$



this could give a larger result!



$P(3) = 200$

$P(7) = 1000$

She should hire 7 employees.