NAME KEY

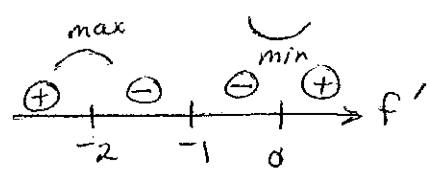
Math 1212
Test 2
Spring 2016

You have 50 minutes to complete this test. You must *show all work* to receive full credit. Work any 7 of the following 8 problems. Clearly **CROSS OUT** the problem you do not wish me to grade. Each problem is worth 14 points, and you get 2 points for free, for a total of 100 points. The answers will be posted on the electronic reserves later today.

1. List the intervals where $f(x)=\dfrac{x^2}{x+1}$ is increasing and the intervals where it is decreasing. Also, find all maximum and minimum points.

$$f'(x) = \frac{2x(x+1)-(x^2)(1)}{(x+1)^2} = \frac{2x^2+2x-x^2}{(x+1)^2} = \frac{x^2+2x}{(x+1)^2}$$

$$f'(x) = \frac{x(x+2)}{(x+1)^2}$$



critical numbers : $x=0, -1, -2$

f is increasing on $(-\infty,-2)\cup(0,\infty)$
f is decreasing on $(-2,-1)\cup(-1,0)$
maximum at $(-2,-4)$
minimum at $(0,0)$

2. For the following functions, find all horizontal and vertical asymptotes (remember that an asymptote is a LINE, not a number). If there are no asymptotes, say so.

a) $f(x)=\dfrac{x(x-2)}{3x^2+7} = \dfrac{x^2-2x}{3x^2+7}$

VA : $3x^2+7=0$
none
HA : $y=1/3$

b) $f(x)=\dfrac{3x+3}{x^2-x-2} = \dfrac{3(x+1)}{(x-2)(x+1)}$

VA : $x=2$ (notice $x=-1$ gives a hole)
HA : $y=0$

c) $f(x)=\sqrt{x} = \dfrac{x^{1/2}}{x^0}$

VA : none
HA : none

3. Suppose $q(p) = \sqrt{2500 - p}$ units of a product are demanded when price is p dollars per unit.

a) Calculate the price elasticity of demand when $p = 1600$. At this price, is the demand elastic or inelastic?

$$E(p) = \frac{p}{q} \cdot q' = \frac{p}{\sqrt{2500-p}} \cdot \frac{1}{2}(2500-p)^{-1/2}(-1) = \frac{-p}{2(2500-p)}$$

$$E(1600) = \frac{-1600}{2(900)} = \frac{-16}{18} = -\frac{8}{9} \leftarrow \text{elasticity}$$

$$|E(1600)| = |-8/9| = \underset{<1}{8/9},$$ so demand is inelastic at this price

b) Write a sentence explaining the meaning of your answer in (a) in plain language. Be as specific as possible.

If price goes up 1% (from \$1600 to \$1616), demand will go down 8/9 % (from $q = \sqrt{2500-1600} = 30$ to $q = 29.7\bar{3}$).

c) Give an example of a product in the correct price range that might behave this way.

Inelastic means it's a necessity. For \$1600, maybe it's a house payment, an emergency trip, medical bill, car repairs...

4. Find the derivatives of the following functions:

a) $f(x) = \dfrac{4x^2 - 5}{(3x^4 + 1)^2}$

$$f'(x) = \frac{(8x)(3x^4+1)^2 - (4x^2-5)(2)(3x^4+1)(12x^3)}{(3x^4+1)^2}$$

b) $f(x) = (2x-9)^4 \sqrt{6x+7}$

$$f'(x) = 4(2x-9)^3(2)\sqrt{6x+7} + (2x-9)^4\left(\tfrac{1}{2}\right)(6x+7)^{-1/2}(6)$$

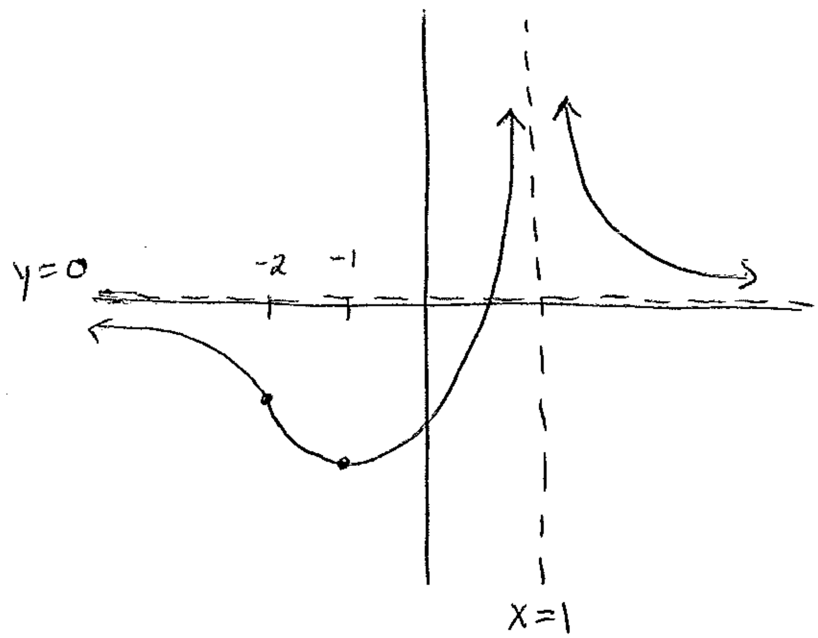
5. Sketch a nice BIG graph of a function with all the properties listed below. Make sure your graph is clearly labeled.

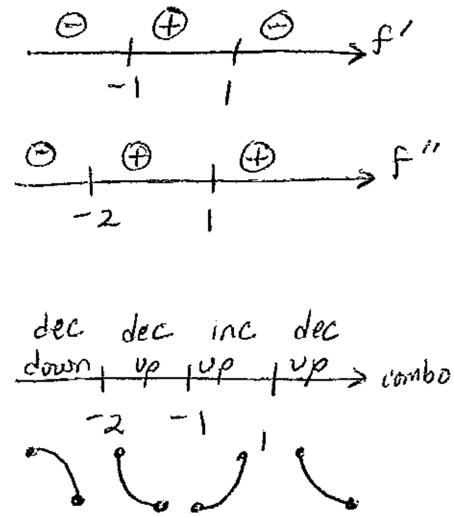
(a) $f'(x) > 0$ when $-1 < x < 1$, but $f'(x) \le 0$ otherwise

(b) $f''(x) > 0$ when $-2 < x < 1$ and when $1 < x$, but $f''(x) \le 0$ otherwise

(c) $f(x)$ is defined for all x except $x = 1$ hole or asymptote

(d) $\lim_{x \to -\infty} f(x) = 0$. HA $y=0$, on left side





6. Find the equation of the line tangent to $x^3y^3 + x = 9$ at the point $(1,2)$.

$$(x^3)(y^3) + x = 9$$

$$(3x^2)(y^3) + (x^3)(3y^2y') + 1 = 0$$

$$3x^3y^2y' = -1 - 3x^2y^3$$

Fill in $x=1, y=2$

$$3(1)(4)\,y' = -1 - 3(1)(8)$$

$$12y' = -1 - 24$$

$$y' = -25/12 = m$$

Line : $y - 2 = \frac{-25}{12}(x-1)$

7. Find the absolute minimum and absolute maximum *points* of $f(x)=3x^4-6x^2$ on the interval $[0,2]$. If such points do not exist, say so and explain why not.

$$f'(x) = 12x^3 - 12x$$
$$= 12x(x^2-1)$$
$$= 12x(x+1)(x-1)$$

<u>CN</u> : $x = 0, -1, 1$

not in interval!

Check CNs <u>and</u> endpoints:

$f(0) = 0$
$f(1) = -3$
$f(2) = 48 - 24 = 24$

absolute max $(2, 24)$
absolute min $(1, -3)$

8. Ms. Jones owns a small insurance agency that sells policies for a large insurance company. For each policy sold, Ms. Jones (who does not sell policies herself) is paid a commission of \$50 by the insurance company. Ms. Jones has determined that when she hires x salespeople, $q(x)=x^3-12x^2+60x$ policies will be sold each week. She pays each salesperson \$750 per week, and her weekly fixed costs are \$2500. Current office facilities can accommodate at most seven salespeople. Determine the number of salespeople Ms. Jones should hire in order to maximize her weekly profit. You must use calculus to solve this problem; "guess and test" methods will not receive credit.

Profit = revenue − cost
= 50(# policies) − 750(# workers) − 2500

$$P(x) = 50(x^3 - 12x^2 + 60x) - 750x - 2500$$
$$= 50x^3 - 600x^2 + 2250x - 2500$$
$$P'(x) = 150x^2 - 1200x + 2250$$
$$= 150(x^2 - 8x + 15)$$
$$= 150(x-3)(x-5)$$

Hire seven salespeople.

<u>one way</u> :

P': + − +

0 3 5

max min

<u>another way</u> :

$P''(x) = 300x - 1200$
$P''(3) = 900 - 1200 < 0$ max
$P''(5) = 1500 - 1200 > 0$ min

Looks like $x = 3$ gives max. Remember to check <u>endpoints</u>, $x = 0$, $x = 7$. $P(0) = -2500$, $\boxed{P(7) = 1000}$ $P(3) = 200$