NAME KEY

Math 1212
Test 3
Fall 2016

You have 50 minutes to complete this test. You must *show all work* to receive full credit. Work any 6 of the following 7 problems. Clearly **CROSS OUT** the problem you do not wish me to grade. Each problem is worth 16 points, and you get 4 points for free, for a total of 100 points. The answers will be posted on the electronic reserves later today.

1. Solve $e^{2y}y' - x^2 = 0$ if $y = 0$ when $x = 0$.

$$e^{2y}y' - x^2 = 0$$

$$\frac{e^{2y}dy}{dx} = x^2$$

$$e^{2y}dy = x^2dx$$

$$\int e^{2y}dy = \frac{1}{3}x^3 + C$$

$$\frac{1}{2}e^{2y} = \frac{1}{3}x^3 + C$$

Fill in $x=0$, $y=0$ to find C.

$$\frac{1}{2}e^0 = \frac{1}{3}(0) + C$$

$$\frac{1}{2} = C$$

$$\frac{1}{2}e^{2y} = \frac{1}{3}x^3 + \frac{1}{2}$$

$$e^{2y} = \frac{2}{3}x^3 + 1$$

$$2y = \ln\left(\frac{2}{3}x^3 + 1\right)$$

$$y = \frac{1}{2}\ln\left(\frac{2}{3}x^3 + 1\right)$$

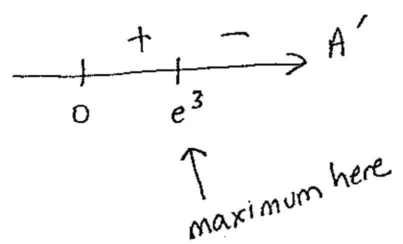
2. The aerobic rating of a person x years old is given by $A(x) = 110\frac{(\ln x - 2)}{x}$ if $x \geq 10$. Find the age when a person's aerobic rating is a maximum.

$$A'(x) = 110\,\frac{\frac{1}{x}(x) - (\ln x - 2)(1)}{x^2} = \frac{1 - \ln x + 2}{x^2} = \frac{3 - \ln x}{x^2} = 0$$

$\ln x = 3$

$x = e^3$ or $x = 0$

$\underline{CN}$: $x = e^3, 0$



Maximum aerobic rating is when a person is $e^3 \approx$ 20.086 years old.

3. Suppose you have $5000 to invest as a single lump sum. You have a choice of investing it either in (a) an account that has an 8.5% annual interest rate compounded quarterly, or (b) an account that has an 8.2% annual interest rate compounded continuously. At the end of your investment, you want to have an ending balance of $10000. Which account will achieve your goal faster?

$B = Pe^{rt}, \quad B = P\left(1+\frac{r}{K}\right)^{Kt}$

a) $10000 = 5000\left(1+\frac{0.085}{4}\right)^{4t}$

$2 = (1.02125)^{4t}$

$\ln 2 = 4t \ln 1.02125$

$\frac{\ln 2}{4\ln 1.02125} = t \approx 8.24$ years

b) $10000 = 5000e^{0.082t}$

$2 = e^{0.082t}$

$\ln 2 = 0.082t$

$\frac{\ln 2}{0.082} = t \approx 8.45$ years

Account (a) will reach $10,000 faster

4. Objects that are heating up or cooling down follow Newton's Law of Cooling. One specific example of this law you can use for this problem is $T(t) = 70 - Ae^{-kt}$, where T is the temperature of the hot liquid, the outside air is 70°F, A and k are constants, and t is the length of time in minutes. Over the upcoming cold weekend, you plan to have a nice hot cup of cocoa. When you first make the cocoa, it's 212°F (from the boiling water), and it is far too hot to drink. After 5 minutes, the cocoa has cooled to 160°F, but it is still a little too hot. If the ideal drinking temperature is 140°F, how long will you have to wait after you make the cocoa to drink it?

① $t=0$ $\quad T=212$

② $t=5$ $\quad T=160$

③ $t=?$ $\quad T=140$

① $212 = 70 - Ae^{0}$

$A = -142$

new, better equation:

$T = 70 + 142e^{-Kt}$

② $160 = 70 + 142e^{-5K}$

$90 = 142e^{-5K}$

$0.6338 = e^{-5K}$

$\ln 0.6388 = -5K$

$K = \frac{\ln 0.6388}{-5} \approx 0.0912$

even better equation:

$T = 70 + 142e^{-0.0912t}$

③ $140 = 70 + 142e^{-0.0912t}$

$70 = 142e^{-0.0912t}$

$0.49296 = e^{-0.0912t}$

$\ln 0.49296 = -0.0912t$

$\frac{\ln 0.49296}{-0.0912} = t \approx 7.756$ minutes

5. a) Solve for x: $-1 = 2\ln x - \frac{1}{3}\ln x$.

$-1 = \ln x^2 - \ln \sqrt[3]{x}$

$-1 = \ln\left(\frac{x^2}{x^{1/3}}\right) = \ln x^{5/3}$

$e^{-1} = x^{5/3}$

$e^{-3/5} = x$

b) Solve for x (simplify): $x = \log_8 \frac{1}{32}$.

$x = \log_8 \frac{1}{32}$

$8^x = \frac{1}{32}$

$2^{3x} = 2^{-5}$

$3x = -5$

$x = -5/3$

6. Evaluate $\int x^2 \ln x \, dx$.

$u = \ln x$ $\qquad dv = x^2 dx$

$du = \frac{1}{x} dx$ $\qquad v = \int x^2 dx = \frac{1}{3} x^3$

$\int u\,dv = uv - \int v\,du$

$\int x^2 \ln x\,dx = (\ln x)(\frac{1}{3}x^3) - \int (\frac{1}{3}x^3)(\frac{1}{x}dx)$

$= \frac{1}{3}x^3 \ln x - \frac{1}{3}\int x^2 dx$

$= \frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + C$

7. For the function $f(x) = xe^{-2x}$, list all intervals of increase and decrease, all maximum and minimum *points*, intervals where the function is concave up and concave down, all inflection *points*, and all asymptotes (or say there are none). Then sketch the graph of the function.

$$f'(x) = e^{-2x} - 2xe^{-2x}$$
$$= e^{-2x}(1-2x) = 0$$

CN: $x = 1/2$ f': $+$ before $1/2$, $-$ after $1/2$

$$f''(x) = -2e^{-2x} - 2e^{-2x} + 4xe^{-2x}$$
$$= -4e^{-2x} + 4xe^{-2x}$$
$$= 4e^{-2x}(-1+x) = 0$$

IN: $x = 1$ f'': $-$ before 1, $+$ after 1

VA: $f(x)$ is defined for all x, so none

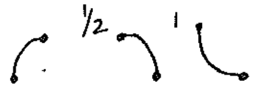
HA: see what happens as $x \to \infty$, $x \to -\infty$

If $x \to \infty$, $y = \text{big} \cdot e^{-\text{big}} = \dfrac{\text{big}}{e^{\text{big}}} \to 0$

HA $y = 0$

If $x \to -\infty$, $y = -\text{big}\, e^{+\text{big}} \to -\infty$

inc down	dec down	dec up
$\frac{1}{2}$	1	combo



Results

increasing on $(\frac{1}{2}, \infty)$

decreasing on $(-\infty, \frac{1}{2})$

maximum $(\frac{1}{2}, \frac{1}{2}e^{-1})$

concave up on $(1, \infty)$

Concave down on $(-\infty, 1)$

inflection point $(1, e^{-2})$

VA: None

HA: $y = 0$