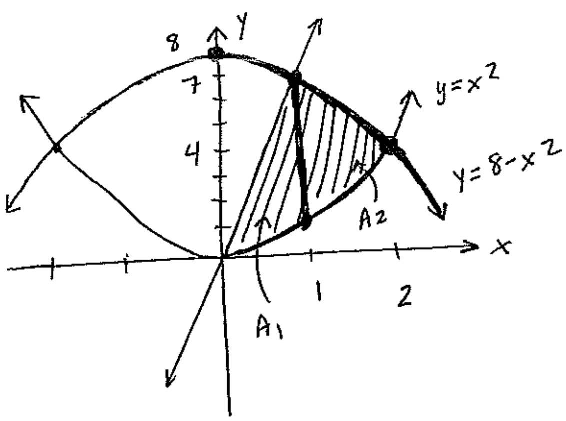
NAME KEY

Math 1212
Test 4
Spring 2016

You have 50 minutes to complete this test. You must *show all work* to receive full credit. Work any 6 of the following 7 problems. Clearly **CROSS OUT** the problem you do not wish me to grade. Each problem is worth 16 points, and you get 4 points for free, for a total of 100 points. The answers will be posted on the electronic reserves later today.

1. Find the area of the region bounded by the curves $y = 8 - x^2$, $y = x^2$, and $y = 7x$ in the first quadrant. Be sure to sketch a graph first! The region should use all three functions as its edges, and only be located in the first quadrant.



$$A = A_1 + A_2$$
$$= \int_0^1 (7x - x^2)\,dx + \int_1^2 (8 - x^2 - x^2)\,dx$$
$$= \left[\frac{7}{2}x^2 - \frac{1}{3}x^3\right]_0^1 + \left[8x - \frac{2}{3}x^3\right]_1^2$$
$$= \left(\frac{7}{2} - \frac{1}{3}\right) - (0 - 0) + \left[\left(16 - \frac{16}{3}\right) - \left(8 - \frac{2}{3}\right)\right]$$
$$= \frac{7}{2} - \frac{1}{3} + 16 - \frac{16}{3} - 8 + \frac{2}{3}$$
$$= 8 + \frac{7}{2} - \frac{15}{3} = 3 + \frac{7}{2} = \frac{13}{2} = 6.5$$

2. For $f(x, y) = 8x^3 + 2x^2y^2 + 5y^4$, show that $f_{xy}(x, y) = f_{yx}(x, y)$.

$f_x = 24x^2 + 4xy^2$ $\qquad$ $f_y = 4x^2y + 20y^3$

$f_{xy} = 8xy$ $\qquad$ $f_{yx} = 8xy$

equal!

3. Find and classify the critical points of $f(x,y) = x^3 + y^3 - xy$.

$f_x = 3x^2 - y = 0 \rightarrow y = 3x^2 \rightarrow$ $3(3x^2)^2 - x = 0$

$f_y = 3y^2 - x = 0$ $3(9x^4) - x = 0$

$x(27x^3 - 1) = 0$

Critical Points: $(0,0), (\frac{1}{3}, \frac{1}{3})$ $x = 0$ or $x^3 = \frac{1}{27}$

$y = 0$ $x = \frac{1}{3}$

$f_{xx} = 6x$ $y = 3(\frac{1}{3})^2 = \frac{1}{3}$

$f_{yy} = 6y$

$f_{xy} = -1$

$D(x,y) = 36xy - 1$

$D(0,0) = -1 < 0$, so $(0,0)$ gives a saddle point.

$D(\frac{1}{3}, \frac{1}{3}) = 36(\frac{1}{9}) - 1 > 0$

$f_{xx}(\frac{1}{3}, \frac{1}{3}) = 6(\frac{1}{3}) > 0$, so $(\frac{1}{3}, \frac{1}{3})$ gives a minimum.

4. Suppose p_1 and p_2 are the prices of two products. Also suppose $D_1(p_1, p_2) = 1000 - 50p_1 + 2p_2$ and $D_2(p_1, p_2) = 500 + 4p_1 - 20p_2$ are the demand functions for the two products (quantities). Answer the following questions, showing your work below.

a) If the price of product 1 goes up by a dollar, the demand for product 2 will go up/down (circle one) by ___4___ units.

b) If the price of product 2 goes up by a dollar, the demand for product 1 will go up/down (circle one) by ___2___ units.

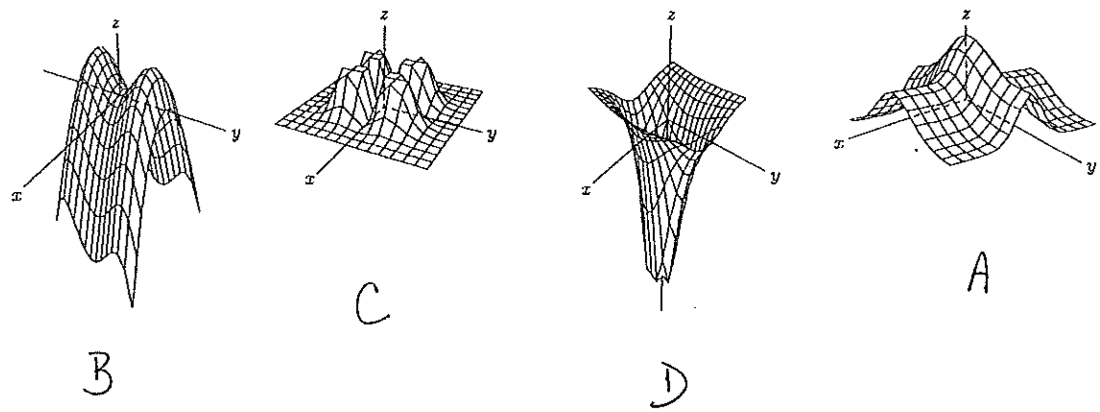
c) These two products are competitive/complementary/neither (circle one). An example of two products that might behave this way are ___Coke___ and ___Pepsi___.

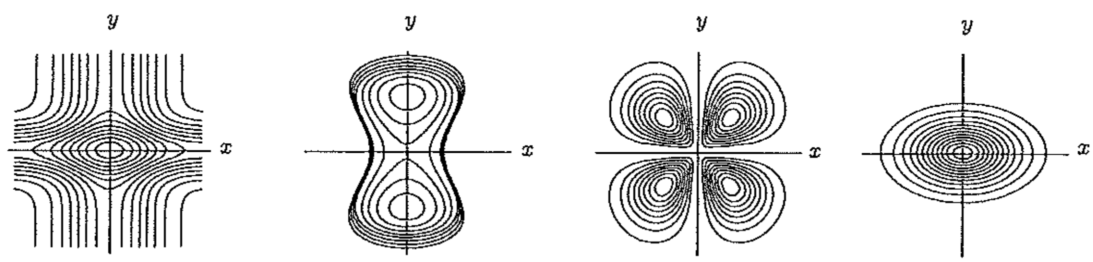
a) $\frac{\partial D_2}{\partial p_1} = 4 > 0$

products are competitive

b) $\frac{\partial D_1}{\partial p_2} = 2 > 0$

5. For each three-dimensional surface below, determine the matching set (a, b, c, or d) of level curves in the xy-plane.





(a) (b) (c) (d)

6. Calculate $\int_1^\infty e^{1-x}\,dx$.

Let $u = 1-x$
$du = -dx$
$-du = dx$

$$\int_1^\infty e^{1-x}\,dx = \lim_{n\to\infty} \int_1^n e^{1-x}\,dx$$

$$= \lim_{n\to\infty} \int_{x=1}^{x=n} e^{u}(-du)$$

$$= \lim_{n\to\infty} \left[-e^{u}\right]_{x=1}^{x=n}$$

$$= \lim_{n\to\infty} \left[-e^{1-x}\right]_1^n = \lim_{n\to\infty} \left(-e^{1-n} + e^{1-1}\right)$$

$$= \lim_{n\to\infty} \left(-e^{1-n} + 1\right) = -e^{\text{big negative}} + 1$$

$$\searrow 0$$

$$= 1$$

7. Suppose a firm has an order for 200 units of its product and wishes to distribute its manufacture between two plants. Suppose x units will be produced at the Minneapolis location and y units will be produced at the Chicago location. If the total cost function is given by $C(x,y) = 2x^2 + xy + y^2 + 200$, how many units should be produced at each location in order to minimize cost?

$x + y = 200$ ← constraint

$C = 2x^2 + xy + y^2 + 200$ ← objective function

$F(x,y,\lambda) = 2x^2 + xy + y^2 + 200 - \lambda(x + y - 200)$

$F_x = 4x + y - \lambda = 0 \longrightarrow y = \lambda - 4x$

$F_y = x + 2y - \lambda = 0 \longrightarrow x + 2(\lambda - 4x) - \lambda = 0$

$-7x + \lambda = 0$

$\lambda = 7x$

$F_\lambda = -x - y + 200 = 0$

$y = 7x - 4x = 3x$

$-x - 3x + 200 = 0$

$200 = 4x$

$50 = x$

$y = 150$

Produce 50 units in Minneapolis and 150 units in Chicago to minimize cost.