NAME KEY

Math 1212
Test 3
Summer 2015

You have 60 minutes to complete this test. You must *show all work* to receive full credit. Work any 7 of the following 8 problems. Clearly **CROSS OUT** the problem you do not wish me to grade. Each problem is worth 14 points, and you get 2 points for free, for a total of 100 points. The answers will be posted on the electronic reserves later today.

1. Solve $\frac{dy}{dx} = e^y \sqrt{x}$ for y.

$$\frac{dy}{dx} = e^y x^{1/2}$$
$$dy = e^y x^{1/2} dx$$
$$e^{-y} dy = x^{1/2} dx$$
$$\int e^{-y} dy = \int x^{1/2} dx$$
$$-e^{-y} = \frac{2}{3} x^{3/2} + C$$

$$\frac{1}{e^y} = -\frac{2}{3} x^{3/2} - C$$
$$e^y = \frac{1}{-\frac{2}{3} x^{3/2} - C}$$
$$y = \ln\left(\frac{1}{-\frac{2}{3} x^{3/2} - C}\right)$$

2. Evaluate $\int x^2 \ln x \, dx$.

$u = \ln x \qquad dv = x^2 dx$

$du = \frac{1}{x} dx \qquad v = \int x^2 dx = \frac{1}{3} x^3$

$$\int u\,dv = uv - \int v\,du$$

$$\int x^2 \ln x \, dx = (\ln x)\left(\frac{1}{3}x^3\right) - \int \left(\frac{1}{3}x^3\right)\left(\frac{1}{x} dx\right)$$
$$= \frac{1}{3}x^3 \ln x - \frac{1}{3}\int x^2 dx$$
$$= \frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + C$$

3. Once the initial publicity surrounding the release of the new *Minions* movie is over, ticket sales will decrease exponentially. At the time publicity is discontinued, suppose the film will have experienced ticket sales of 1,500,000 per month. One month later, suppose sales drop to 1,000,000 per month. What will sales be six months after publicity is discontinued?

$B = Pe^{rt}$

① $t=0$ $\quad B = 1500000$

② $t=1$ $\quad B = 1000000$

③ $t=6$ $\quad B = ?$

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① $1500000 = Pe^{r(0)} = P$

Now $B = 1500000e^{rt}$.

② $1000000 = 1500000e^{r(1)}$

$\frac{1000000}{1500000} = e^r$

$\frac{2}{3} = e^r$

$r = \ln 2/3 \approx -0.4055$

Now $B = 1500000e^{-0.4055t}$

③ $B = 1500000e^{-0.4055(6)}$

$B \approx 131,687.24$

so about $\boxed{131,687}$ tickets will be sold 6 months after promotion ends.

4. Solve the following integrals:

(a) $\int \frac{x^2+3x-2}{\sqrt{x}}\,dx$

$$= \int (x^2+3x-2)(x^{-1/2})\,dx$$
$$= \int (x^{3/2} + 3x^{1/2} - 2x^{-1/2})\,dx$$
$$= \frac{2}{5}x^{5/2} + 3\cdot\frac{2}{3}x^{3/2} - 2\cdot 2x^{1/2} + C$$
$$= \frac{2}{5}x^{5/2} + 2x^{3/2} - 4x^{1/2} + C$$

(b) $\int \frac{2x^4}{x^5+1}\,dx$

Let $u = x^5+1$

then $du = 5x^4\,dx$

$\frac{1}{5}du = x^4dx$

$$= \int \frac{2(\frac{1}{5}du)}{u} = \frac{2}{5}\int \frac{1}{u}\,du$$
$$= \frac{2}{5}\ln|u| + C = \frac{2}{5}\ln|x^5+1| + C$$

5. The rate at which a student employee can file papers is a function of the employee's experience. It is estimated that after t weeks on the job, the average student employee can file $Q(t) = 700 - 400e^{-0.5t}$ papers per hour.

a) How many papers can a new employee file per hour?

If $t=0$, $Q = 700 - 400e^{0}$
$= 700 - 400$
$= 300$ papers per hour

b) How many papers can a student employee with 6 weeks experience file per hour?

If $t=6$, $Q = 700 - 400e^{-3} \approx 680$ papers per hour

c) Approximately how many papers will a student employee be able to file per hour after an extended period of employment? (Show work and explain!)

extended period of employment means $t \to \infty$.

$700 - 400e^{-0.5t} \to 700 - 400\left(\frac{1}{e^{\text{big}}}\right)$
$\to 700 - 400$ (very small #, close to 0)
$\to 700$ papers per hour

6. The marginal profit of a certain company is $P'(q) = 100 - 2q$ dollars when q units are produced. If the company's profit is \$700 when 10 units are produced, what is the company's maximum profit?

$P' = 100 - 2q =$ marginal profit

$P = \int (100 - 2q)\,dq$

$P = 100q - q^2 + C =$ profit

$700 = 100(10) - (10)^2 + C = 1000 - 100 + C$

$C = -200$

$P = 100q - q^2 - 200 =$ profit

For maximum, $P' = 0$, so $100 - 2q = 0$

CN: $q = 50$

$\xrightarrow[50]{+ \quad | \quad -} P'$ so $q = 50$ will give maximum.

maximum profit is

$P(50) = 100(50) - (50)^2 - 200$
$= \$2300$

7. a) If $5 = 3\ln x - \frac{1}{2}\ln x$, solve for x. Your answer should be exact, not in decimal form.

$$5 = \ln x^3 - \ln x^{1/2}$$
$$5 = \ln\left(\frac{x^3}{x^{1/2}}\right)$$
$$5 = \ln x^{5/2}$$
$$5 = \frac{5}{2}\ln x$$
$$2 = \ln x$$
$$\boxed{x = e^2}$$

alternately:

$$\longrightarrow e^5 = x^{5/2}$$
$$(e^5)^{2/5} = x$$
$$e^2 = x$$

b) If $\log_3 x = 2$, $\log_3 y = 3$, and $\log_3 z = 4$, find $\log_3 \frac{x^3}{y\sqrt{z}}$. Your answer should be a number.

$$\log_3\left(\frac{x^3}{yz^{1/2}}\right) = \log_3 x^3 - \left(\log_3 y + \log_3 z^{1/2}\right)$$
$$= 3\log_3 x - \log_3 y - \frac{1}{2}\log_3 z$$
$$= 3(2) - 3 - \frac{1}{2}(4)$$
$$= 6 - 3 - 2$$
$$= 1$$

8. Find the equation of the line tangent to $f(x) = \ln(2x-1) + 4e^{6x-6}$ at the point where $x = 1$.

<u>point</u>: $x = 1$, $y = \ln(1) + 4e^0 = 0 + 4 = 4$

$(1, 4)$

<u>slope</u>: $f'(x) = \frac{1}{2x-1}\cdot 2 + 4e^{6x-6}\cdot 6 = \frac{2}{2x-1} + 24e^{6x-6}$

$m = f'(1) = \frac{2}{2-1} + 24e^0 = 2 + 24 = 26$

<u>Line</u>: $y - 4 = 26(x-1)$

$y = 26x - 22$