NAME KEY

Math 12
Test 3
Spring 2014

You have 50 minutes to complete this test. You must *show all work* to receive full credit. Work any 7 of the following 8 problems. Clearly **CROSS OUT** the problem you do not wish me to grade. Each problem is worth 14 points, and you get 2 points for free, for a total of 100 points. The answers will be posted on the electronic reserves later today.

1. Solve $\frac{dy}{dx} = \frac{x^3 + 2x - 7}{2xy}$ for y.

$$2y\,dy = \frac{x^3+2x-7}{x}\,dx$$

$$\int 2y\,dy = \int \frac{x^3+2x-7}{x}\,dx$$

$$y^2 = \int\left(\frac{x^3}{x} + \frac{2x}{x} - \frac{7}{x}\right)dx = \int\left(x^2 + 2 - \frac{7}{x}\right)dx$$

$$y^2 = \frac{1}{3}x^3 + 2x - 7\ln|x| + C$$

$$y = \pm\sqrt{\frac{1}{3}x^3 + 2x - 7\ln|x| + C}$$

2. Evaluate $\int \frac{(\ln x)^2}{x}\,dx$.

Let $u = \ln x$.
Then $du = \frac{1}{x}\,dx$
$x\,du = dx$

$$\int \frac{(\ln x)^2}{x}\,dx = \int \frac{u^2}{x}(x\,du)$$

$$= \int u^2\,du$$

$$= \frac{1}{3}u^3 + C$$

$$= \frac{1}{3}(\ln x)^3 + C$$

3. Find all maxima, minima and inflection points of $f(x) = 4 + e^{2x}$. Also give the intervals where f is increasing, decreasing, concave up, and concave down. Find all vertical and horizontal asymptotes, or state that none exist. Then carefully sketch the graph of f.

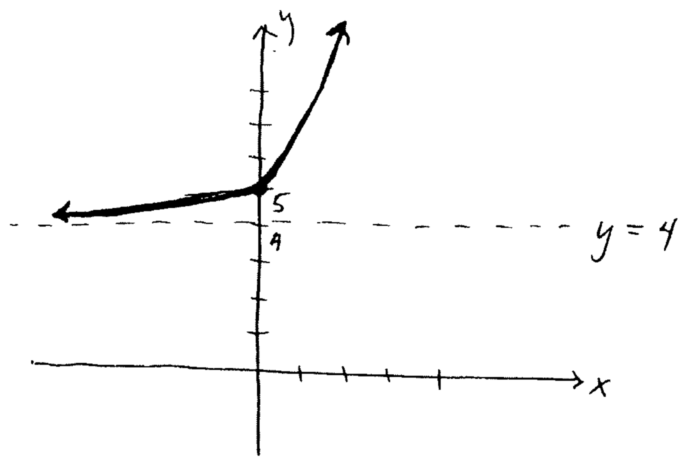
$f'(x) = 2e^{2x} \neq 0$ no critical numbers $\xrightarrow{\quad + \quad} f'$

$f''(x) = 4e^{2x} = 0$ no inf. pts $\xrightarrow{\quad + \quad} f''$

$\left.\begin{array}{l} \text{If } x \to \infty,\ f(x) = 4 + e^{2x} \to 4 + e^{\text{big}+} \to \infty \\ \text{If } x \to -\infty,\ f(x) = 4 + e^{2x} \to 4 + e^{\text{big}-} \to 4 + 0 \to 4 \end{array}\right\}$ horiz asymp tests

$f(x) = 4 + e^{2x}$ is defined everywhere $\}$ vert asymp. test

no max or min
no inf. pts
inc. on $(-\infty, \infty)$
never dec.
conc. up on $(-\infty, \infty)$
never conc down
no vertical asymp
horiz. asymp $y = 4$



4. Find $f'(x)$ for the following functions. DO NOT simplify!

(a) $f(x) = x^2 \ln(3x+1)$

$$f'(x) = 2x \ln(3x+1) + (x^2)\left(\frac{1}{3x+1}\right)(3)$$

(b) $f(x) = e^{\sqrt{4x^3+1}}$

$$f'(x) = \left(e^{\sqrt{4x^3+1}}\right)\left(\frac{1}{2}(4x^3+1)^{-1/2}(12x^2)\right)$$

5. Joe turns 18 in May, at which time he will gain control of a small trust fund meant to pay for his education and various other expenses. Joe wants to budget his money so that in 4 years when he graduates, he can purchase a new car. He estimates that the new car will cost $25,000. How much should he put aside now out of the trust fund money so that he can purchase the car when he graduates? Assume that he can earn an annual interest rate of 6% compounded quarterly.

$$B = P\left(1+\frac{r}{k}\right)^{kt}$$

$$25000 = P\left(1+\frac{0.06}{4}\right)^{4(4)}$$

$$25000 = P(1.015)^{16}$$

$$\frac{25000}{1.015^{16}} = P$$

$$P \approx \frac{25000}{1.26898} \approx \$19700.78$$

6. The population density x miles from a remote city out west is given to be $D(x) = 12e^{-0.07x}$ thousand people per square mile. How far from the center of the city will the population density be 30 people per square mile?

$$30 \text{ people} = \frac{30}{1000} \text{ "thousand" people}$$

$$= 0.03 \text{ thousand people}$$

$$0.03 = 12e^{-0.07x}$$

$$0.0025 = e^{-0.07x}$$

$$\ln 0.0025 = -0.07x$$

$$\frac{\ln 0.0025}{-0.07} = x$$

$x \approx 85.6$ miles from the center of the city

7. a) Rewrite $\ln 3 + 2\ln x - \frac{1}{2}\ln y$, as an expression containing a single logarithm.

$$\ln 3 + 2\ln x - \tfrac{1}{2}\ln y = \ln 3 + \ln x^2 - \ln\sqrt{y}$$
$$= \ln 3x^2 - \ln\sqrt{y}$$
$$= \ln\left(\frac{3x^2}{\sqrt{y}}\right)$$

b) Suppose $\log_2 x = 4$, $\log_2 y = 3$, and $\log_2 z = 2$. Find $\log_2 \frac{xy^2}{z^3}$.

$$\log_2 \frac{xy^2}{z^3} = \log_2 xy^2 - \log_2 z^3$$
$$= \log_2 x + \log_2 y^2 - \log_2 z^3$$
$$= \log_2 x + 2\log_2 y - 3\log_2 z$$
$$= 4 + 2(3) - 3(2)$$
$$= 4$$

8. Evaluate $\int 3x(2x+1)^6\,dx$.

method 1 :

Let $u = 2x+1$. $\rightarrow x = \frac{u-1}{2}$

$du = 2dx$

$\frac{1}{2}du = dx$

$$\int 3x(2x+1)^6 dx = \int 3\left(\frac{u-1}{2}\right)u^6\left(\tfrac{1}{2}du\right)$$
$$= \frac{3}{4}\int(u^7 - u^6)\,du$$
$$= \frac{3}{4}\left(\frac{(2x+1)^8}{8} - \frac{(2x+1)^7}{7}\right) + C$$

method 2 : $\int u\,dv = uv - \int v\,du$

Let $u = 3x$ $\quad dv = (2x+1)^6 dx$

$du = 3dx$ $\quad v = \int (2x+1)^6 dx$ $\quad w = 2x+1$

$= \int w^6\left(\tfrac{1}{2}dw\right)$ $\quad dw = 2dx$

$= \frac{1}{2}\left(\frac{1}{7}\right)(2x+1)^7$ $\quad \frac{1}{2}dw = dx$

$$\int 3x(2x+1)^6 dx = (3x)\left(\tfrac{1}{14}\right)(2x+1)^7 - \int \tfrac{1}{14}(2x+1)^7(3dx)$$
$$= \frac{3}{14}x(2x+1)^7 - \frac{3}{14}\int(2x+1)^7 dx$$
$$= \frac{3}{14}x(2x+1)^7 - \frac{3}{28}\,\frac{(2x+1)^8}{8} + C$$