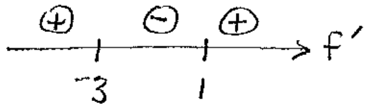
NAME KEY

Math 12
Test 2
Summer 2014

You have 60 minutes to complete this test. You must *show all work* to receive full credit. Work any 7 of the following 8 problems. Clearly **CROSS OUT** the problem you do not wish me to grade. Each problem is worth 14 points, and you get 2 points for free, for a total of 100 points. The answers will be posted on the electronic reserves later today.

1. Suppose $f(x) = 2x^3 + 6x^2 - 18x + 5$. Find all extreme points and list the intervals of increase and decrease. Then list the intervals where the function is concave up and where it is concave down, and find all inflection points.

$$f'(x) = 6x^2 + 12x - 18$$
$$= 6(x^2 + 2x - 3)$$
$$= 6(x+3)(x-1)$$

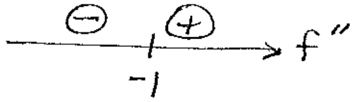


$f(-3) = -54 + 54 + 54 + 5 = 59$

$f(1) = 2 + 6 - 18 + 5 = -5$

$f(-1) = -2 + 6 + 18 + 5 = 27$

$$f''(x) = 12x + 12$$



increasing on $(-\infty, -3) \cup (1, \infty)$

decreasing on $(-3, 1)$

max $(-3, 59)$

min $(1, -5)$

conc up $(-1, \infty)$

conc down $(-\infty, -1)$

inf. pt $(-1, 27)$

2. For the following functions, find all horizontal and vertical asymptotes (remember that an asymptote is a LINE, not a number). If there are no asymptotes, say so.

(a) $f(x) = \dfrac{1\enclose{circle}{-3x^2}}{\enclose{circle}{2x^2} - 4x + 2} = \dfrac{1-3x^2}{2(x^2-2x+1)} = \dfrac{1-3x^2}{2(x-1)^2}$

vertical : $x = 1$

horizontal: $y = -3/2$

(b) $f(x) = \dfrac{\enclose{circle}{x} - 3}{\enclose{circle}{x^2} + x - 12} = \dfrac{x-3}{(x-3)(x+4)}$

vertical: $x = -4$ (notice $x = 3$ is a hole)

horizontal: $y = 0$

(c) $f(x) = \dfrac{(1+x^2)^3}{1}$

vertical: none

horizontal : none

3. Suppose the demand for a product is given by $q(p) = 500 - 2p$ where p is price. When $p = 100$, find the price elasticity of demand. Is demand elastic or inelastic at this price? Give an example of a product that might behave in this way.

$$E(p) = \frac{p}{q} \cdot q' = \frac{p}{500-2p} \cdot (-2)$$

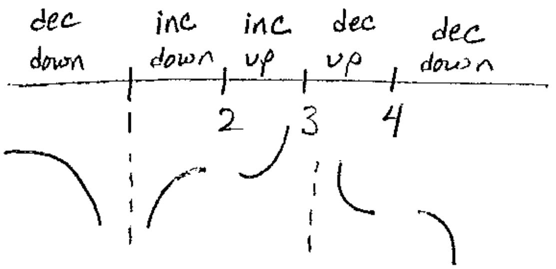
$$E(100) = \frac{100}{500-200} (-2) = \frac{-200}{300} = -2/3$$

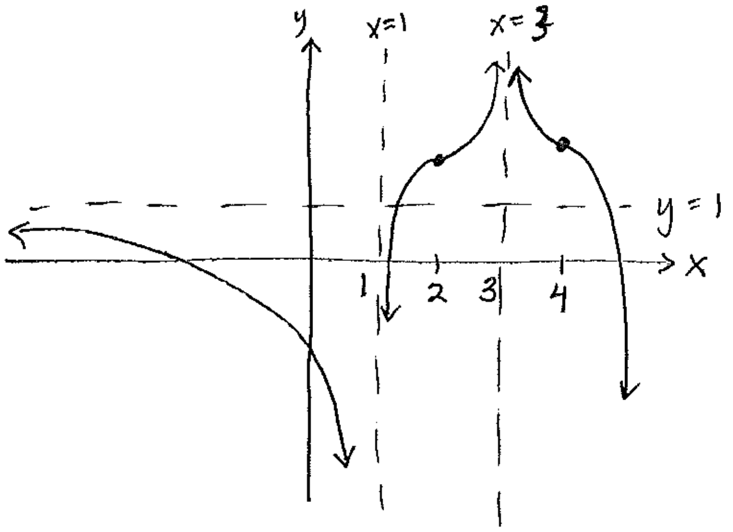
$|E(100)| = 2/3 < 1$, so demand is <u>inelastic</u>.

products that behave this way are considered necessities. Price of \$100 could be products like car repairs, groceries . . .

4. Sketch a nice BIG graph of a function with all the properties listed below. Make sure your graph is clearly labeled.

a) $f'(x) > 0$ when $1 < x < 3$, and $f'(x) \le 0$ otherwise
b) $f''(x) > 0$ when $2 < x < 3$ and when $3 < x < 4$, but $f''(x) \le 0$ otherwise
c) $f(1)$ and $f(3)$ are undefined asymptotes or holes
d) $\lim_{x \to -\infty} f(x) = 1$. ← horiz asymp $y = 1$





5. Find $f'(x)$ for the following functions. DO NOT simplify!

(a) $f(x) = \frac{1-5x^2}{\sqrt{3+2x}} = \frac{1-5x^2}{(3+2x)^{1/2}}$

$$f'(x) = \frac{(-10x)(3+2x)^{1/2} - (1-5x^2)(\frac{1}{2})(3+2x)^{-1/2}(2)}{3 + 2x}$$

(b) $f(x) = x^3(2x^2 + x - 3)^2$

$$f'(x) = 3x^2(2x^2+x-3)^2 + x^3(2)(2x^2+x-3)(4x+1)$$

6. Find y' if $5x - x^2y^3 = 2y$.

$$5 - (2xy^3 + x^2(3y^2y')) = 2y'$$

$$5 - 2xy^3 - 3x^2y^2y' - 2y' = 0$$

$$-3x^2y^2y' - 2y' = 2xy^3 - 5$$

$$y'(-3x^2y^2 - 2) = 2xy^3 - 5$$

$$y' = \frac{2xy^3 - 5}{-3x^2y^2 - 2}$$

7. Find the absolute minimum and absolute maximum *points* of $f(x) = 3x^5 - 5x^3$ on the interval $-2 \le x \le 0$.

$f'(x) = 15x^4 - 15x^2$

$= 15x^2(x^2 - 1)$

$= 15x^2(x+1)(x-1).$ Crit #'s: $x = 0, 1, -1$ ↙ out of range!

$f(0) = 0$ ← abs max

$f(-1) = -3+5 = 2$

$f(-2) = 3(-32) - 5(-8) = -96 + 40 = -56$ ← abs min

absolute max $(-1, 2)$

absolute min $(-2, -56)$

8. Suppose that when the price of a commodity is $p(q) = 37 - 2q$, all q units will be sold. The total cost of producing q units is given by $C(q) = 3q^2 + 5q + 75$. Find the quantity that should be produced in order to maximize profit. (Be sure your answer gives a *maximum*.)

Profit = Revenue - Cost

= price · quantity − Cost

$P(q) = (37 - 2q)(q) - (3q^2 + 5q + 75)$

$= 37q - 2q^2 - 3q^2 - 5q - 75$

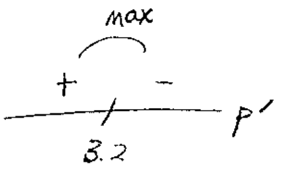
$= -5q^2 + 32q - 75$

$P'(q) = -10q + 32 = 0$

$q = 3.2$

Is it a max?

method ①



method ②

$p''(q) = -10$

$p''(3.2) = -10 < 0$

conc. down

producing 3.2 units will give maximum profit.

(Seems odd to produce .2 units, but this could still make practical sense if q were in, say "hundreds of units").