NAME KEY

Math 1212
Test 3
Spring 2015

You have 50 minutes to complete this test. You must *show all work* to receive full credit. Work any 7 of the following 8 problems. Clearly **CROSS OUT** the problem you do not wish me to grade. Each problem is worth 14 points, and you get 2 points for free, for a total of 100 points. The answers will be posted on the electronic reserves later today. Since we are doing Basic Skills tomorrow, exams will be returned on Wednesday.

1. Solve $\frac{dy}{dx} = 4x^3y^2$ if $y = 2$ when $x = 1$.

$\frac{dy}{y^2} = 4x^3\,dx$

$\int y^{-2}dy = \int 4x^3dx$

$\frac{y^{-1}}{-1} = x^4 + C$

$\frac{1}{y} = -x^4 - C$

If $x=1$, $y=2$, so

$\frac{1}{2} = -1 - C$

$\frac{3}{2} = -C$

$C = -3/2.$

$\frac{1}{y} = -x^4 + \frac{3}{2}$

$y = \frac{1}{-x^4 + \frac{3}{2}}$

2. Evaluate the following.

a) $\int (4x+2)e^{3x^2+3x-1}dx$

$= 2\int (2x+1)e^{3x^2+3x-1}dx$

$u = 3x^2+3x-1$

$du = (6x+3)dx$

$\frac{1}{3}du = (2x+1)dx$

$= 2\int e^u(\frac{1}{3}du)$

$= \frac{2}{3}e^u + C$

$= \frac{2}{3}e^{3x^2+3x-1} + C$

b) $\int \frac{1}{x(\ln x)^2}dx$

$= \int \frac{1}{(\ln x)^2}\frac{1}{x}dx$

$u = \ln x$

$du = \frac{1}{x}dx$

$= \int \frac{1}{u^2}du = \int u^{-2}du$

$= \frac{u^{-1}}{-1} + C$

$= -\frac{1}{\ln x} + C$

3. Find all maxima, minima and inflection points of $f(x) = x \ln x$ for $x>0$. Also give the intervals where f is increasing, decreasing, concave up, and concave down. Find all vertical and horizontal asymptotes, or state that none exist. Then carefully sketch the graph of f.

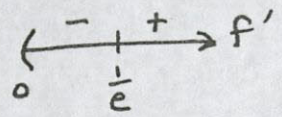
$f(x) = x \ln x$

$f'(x) = \ln x + x\left(\frac{1}{x}\right)$

$= \ln x + 1 = 0$

$\ln x = -1$

CN: $x = e^{-1} = \frac{1}{e}$



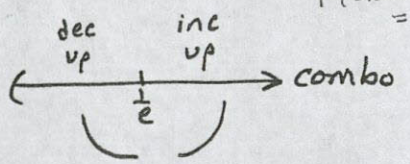
$f''(x) = \frac{1}{x}$ never zero,

no IN.

HA: If $x \to \infty$, $f(x) \to \infty$.

VA: Undef when $x=0$. $f(0.0001) = -0.00092$

f(close to 0) = close to 0



Results

inc $(\frac{1}{e}, \infty)$

dec $(0, \frac{1}{e})$

min $(\frac{1}{e}, \frac{-1}{e})$

no max

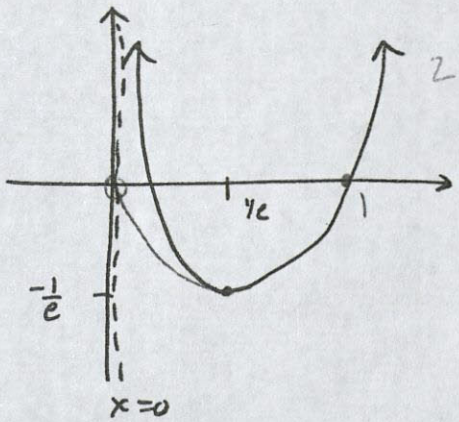
conc up $(-\infty, \infty)$

never conc down

no inf pts

no HA

VA : none (will accept $x=0$)



4. Find $f'(x)$ for the following functions. DO NOT simplify!

(a) $f(x) = x^2 \ln\sqrt{x^2+1}$

$f'(x) = 2x \ln(x^2+1)^{1/2} + x^2 \cdot \frac{1}{\sqrt{x^2+1}} \cdot \frac{1}{2}(x^2+1)^{-1/2}(2x)$

(b) $f(x) = \frac{e^{-3x}}{x^2+1}$

$f'(x) = \frac{-3e^{-3x}(x^2+1) - e^{-3x}(2x)}{(x^2+1)^2}$

5. Which account will earn more money, Account A, earning 6% annual interest compounded monthly, or Account B, earning 5.5% interest compounded continuously?

Suppose you start with P dollars. How much will you have in a year?

(A) $B = P\left(1+\frac{0.06}{12}\right)^{12} = 1.062P$ ← this is bigger

(B) $B = Pe^{0.055} \approx 1.057P$

Account A earns more money.

(Can use a fixed dollar amount for P, and any number of years you like).

6. A family of rabbits has taken up residence under the azalea in my backyard. Currently there are 5 rabbits, and a month from now there will be 8 rabbits. Assuming that rabbits multiply exponentially, and that I keep my cat indoors, how long will it be until there are 100 rabbits?

$B = Pe^{rt}$

$t=0 \quad B=5$

$t=1 \quad B=8$

$t=? \quad B=100$

① $B = Pe^{rt}$

$5 = Pe^{r(0)} = P$

so $B = 5e^{rt}$

② $8 = 5e^{r}$

$1.6 = e^{r}$

$\ln 1.6 = r \approx 0.47$, so $B \doteq 5e^{0.47t}$

③ $100 = 5e^{0.47t}$

$20 = e^{0.47t}$

$\ln 20 = 0.47t$

$t \approx \frac{\ln 20}{0.47} \approx$ **6.37 months**

7. a) If $5 = 1 + 4e^{-6x}$, solve for x.

$4 = 4e^{-6x}$

$1 = e^{-6x}$

$\ln 1 = -6x$

$0 = -6x$

$x = 0$

b) If $\log_3 x = \frac{1}{3}(\log_3 16 + 2\log_3 2)$, solve for x.

$\log_3 x = \frac{1}{3}(\log_3 16 + \log_3 4)$

$= \frac{1}{3}(\log_3 64)$

$= \log_3 (\sqrt[3]{64})$

$= \log_3 4$

$3^{\log_3 x} = 3^{\log_3 4}$

$x = 4$

8. Evaluate $\int \frac{\ln 3x}{x^2}\,dx$.

$$\int \frac{\ln 3x}{x^2}\,dx = \int \ln 3x \cdot \frac{1}{x^2}\,dx$$

Let $u = \ln 3x$ $\qquad dv = \frac{1}{x^2}\,dx$

$du = \frac{1}{3x} \cdot 3\,dx$ $\qquad v = \int x^{-2}\,dx = \frac{x^{-1}}{-1} = -\frac{1}{x}$

$= \frac{1}{x}\,dx$

$$\int \frac{\ln 3x}{x^2}\,dx = uv - \int v\,du = \frac{-1}{x} \cdot \ln 3x - \int \frac{-1}{x} \cdot \frac{1}{x}\,dx$$

$$= \frac{-1}{x}\ln 3x + \int x^{-2}\,dx$$

$$= \frac{-1}{x}\ln 3x - x^{-1} + C$$