You have 60 minutes to complete this test. You must *show all work* to receive full credit. Work any 7 of the following 8 problems. Clearly **CROSS OUT** the problem you do not wish me to grade. Each problem is worth 14 points, and you get 2 points for free, for a total of 100 points. If you have any questions, please come to the front and ask.

1.  Using the *definition* of the derivative, find $f'(x)$ if $f(x) = \sqrt{x-4}$ .

2.  Evaluate the following limits. If any of them do not exist, EXPLAIN why not ("because it's undefined" and "denominator is zero" are not sufficient explanations).

    (a)   $\displaystyle \lim_{x \to 3} \frac{x^2 + 2x + 1}{x + 3}$

    (b)   $\displaystyle \lim_{x \to 4} \frac{x - 4}{\sqrt{x} - 2}$

    (c)   $\displaystyle \lim_{x \to 2^+} \frac{1}{\sqrt{x^2 - 4}}$

3. The total cost of producing $x$ packages of cookies is $C(x) = \frac{1}{20}x^2 + 3x + 33$ dollars. All $x$ packages will be sold if the price is set at $p(x) = \frac{1}{5}(45 - x)$ dollars per package.

    a) Find an equation for profit when $x$ packages of cookies are produced and sold.

    b) *Estimate* the profit gained from the production and sale of the 11th package.
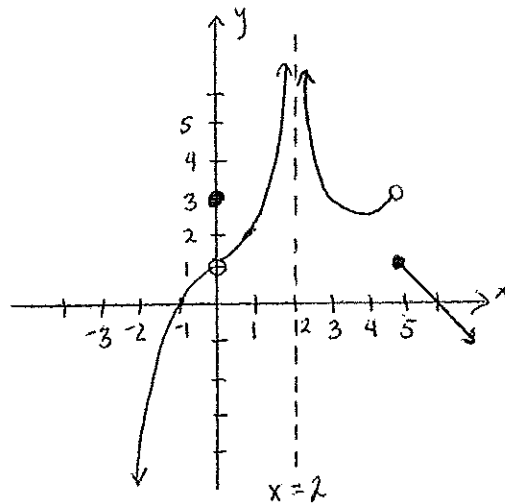
    c) Find the *actual* profit from the 11th package.

4. Find $f'(x)$ (do not simplify!) if :

    a) $f(x) = \dfrac{x^2 - 3x + 2}{2x^2 - 5x + 1}$

    b) $f(x) = -\dfrac{x^2}{16} + \dfrac{2}{x} - x^{\frac{3}{2}} + \dfrac{1}{3x^2} + \dfrac{x}{3}$

5.   Find the equation of the line tangent to the graph of the function $f(x) = (3x+1)(2x^2 - 4)(5x^3 + 2x - 1)$ at the point where $x = 0$.

6.   Consider the graph of the function $f(x)$ given below.



(a)   For what values of $x$ is $f(x)$ discontinuous?

(b)   Find $\lim\limits_{x \to -1} f(x)$.

(c)   Find $\lim\limits_{x \to 0} f(x)$.

(d)   Find $\lim\limits_{x \to 2} f(x)$.

(e)   Find $\lim\limits_{x \to 5^-} f(x)$.

(f)   Find $\lim\limits_{x \to 5^+} f(x)$.

7. Carefully graph the function $f(x) = \begin{cases} 3 & \text{if } x \leq 0 \\ x^2 + 2 & \text{if } 0 < x < 2 \\ -2x + 10 & \text{if } 2 \leq x \end{cases}$. Does this function have any discontinuities, and if so where?

8. A bakery can produce small wedding cakes at a cost of $80 apiece. Sales figures indicate that if the cakes are sold for $x$ dollars each, approximately $300 - x$ cakes will be sold during the May-September wedding season. Find an equation for *profit*, and determine the price and number of cakes that will maximize profit. What will be the maximum profit?