NAME KEY

Math 12
Test 2
Spring 2014

You have 50 minutes to complete this test. You must *show all work* to receive full credit. Work any 7 of the following 8 problems. Clearly **CROSS OUT** the problem you do not wish me to grade. Each problem is worth 14 points, and you get 2 points for free, for a total of 100 points. The answers will be posted on the electronic reserves later today.

1. Suppose $f(x)=\frac{6x^4}{x^2-3}$. Find all intervals where $f(x)$ is increasing and where it is decreasing (interval notation, please). List the extreme points and tell whether they are maxima or minima.

$$f'(x)=\frac{24x^3(x^2-3)-(6x^4)(2x)}{(x^2-3)^2}$$

$$=\frac{24x^5-72x^3-12x^5}{(x^2-3)^2}$$

$$=\frac{12x^5-72x^3}{(x^2-3)^2}$$

$$=\frac{12x^3(x^2-6)}{(x^2-3)^2}$$

CN: $x=0, \pm\sqrt{3}, \pm\sqrt{6}$

f': $\ominus$ $-\sqrt{6}$ $\oplus$ $-\sqrt{3}$ $\oplus$ 0 $\ominus$ $\sqrt{3}$ $\ominus$ $\sqrt{6}$ $\oplus$

inc on $(-\sqrt{6},-\sqrt{3})\cup(-\sqrt{3},0)\cup(\sqrt{6},\infty)$

dec on $(-\infty,-\sqrt{6})\cup(0,\sqrt{3})\cup(\sqrt{3},\sqrt{6})$

max $(0,0)$

min $(\pm\sqrt{6}, 72)$

2. For the following functions, find all horizontal and vertical asymptotes (remember that an asymptote is a LINE, not a number). If there are no asymptotes, say so.

a) $f(x)=\frac{x^3}{x^2-9} = \frac{x^3}{(x+3)(x-3)}$

VA: $x=\pm 3$

HA: none

b) $f(x)=\frac{x^4+1}{1-x^4} = \frac{x^4+1}{(1-x^2)(1+x^2)} = \frac{x^4+1}{(1-x)(1+x)(1+x^2)}$

VA: $x=\pm 1$

HA: $y=-1$

c) $f(x)=\frac{3x+4}{9x^2-16} = \frac{3x+4}{(3x+4)(3x-4)}$

VA: $x=4/3$

(notice $x=-4/3$ gives hole)

HA: $y=0$

3. Suppose that at price p, demand for a certain product is given by $q(p)=\sqrt{2500-p}$.

a) Find the price elasticity of demand when price is \$900.

$$E(p)=\frac{p}{q}\cdot q' = \frac{p}{\sqrt{2500-p}}\cdot\frac{1}{2}(2500-p)^{-1/2}(-1)=\frac{-p}{2(2500-p)}$$

$$E(900)=\frac{-900}{2(1600)}=\boxed{\frac{-9}{32}}$$

b) Is demand elastic or inelastic at this price?

$|E(900)|=\frac{9}{32}<1$, so inelastic.

c) Give an example of a product in the correct price range that might behave as described in (a).

must be a necessity that costs about \$900 ...
House payment, tuition, ...

4. Determine where the function $f(x)=\dfrac{100(x+5)}{x^2}$ is increasing and where it is decreasing, and where it is concave up and concave down. Find all extrema and inflection points. Find all asymptotes. Then sketch the graph, labeling as appropriate.

$$f'(x)=\frac{100x^2-100(x+5)(2x)}{x^4}$$
$$=\frac{100x^2-200x^2-1000x}{x^4}$$
$$=\frac{-100x^2-1000x}{x^4}=\frac{-100x-1000}{x^3}$$
$$=\frac{-100(x+10)}{x^3}$$

CN : $x=0,\ 10$

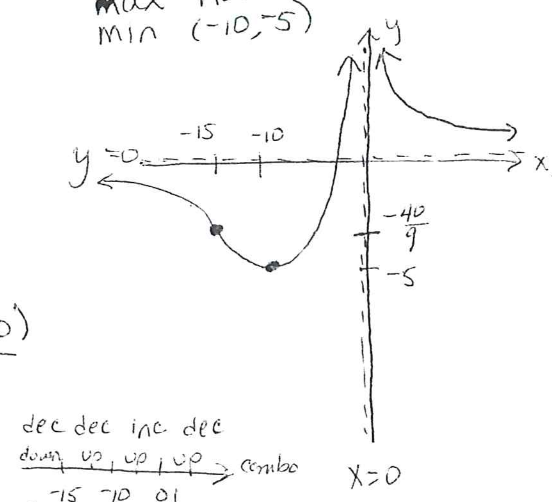
f': $\ominus$ -10 $\oplus$ 0 $\ominus$

inc $(-10,0)$
dec $(-\infty,-10)\cup(0,\infty)$
conc up $(-15,0)\cup(0,\infty)$
conc down $(-\infty,-15)$
HA : $y=0$
VA : $x=0$
max none
min $(-10,-5)$
inf pt $(-15,\ -\frac{40}{9})$

$$f''(x)=\frac{-100x^3-(-100x-1000)(3x^2)}{x^6}$$
$$=\frac{-100x^3+300x^3+3000x^2}{x^6}$$
$$=\frac{200x^3+3000x^2}{x^6}=\frac{100(2x+30)}{x^4}$$

IN : $x=0,-15$

f'': $\ominus$ -15 $\oplus$ 0 $\oplus$



5. Find all absolute extrema of $f(x) = 3x^4 - x^6$ on the interval $[-1, 2]$.

$f'(x) = 12x^3 - 6x^5$

$= 6x^3(2 - x^2)$

CN: $x = 0, \pm\sqrt{2}$

check $x = 0, \sqrt{2}, -1, 2$

notice $x = -\sqrt{2}$ is not in $[-1, 2]$.

$f(0) = 0$

$f(\sqrt{2}) = 12 - 8 = 4$

$f(-1) = 2$

$f(2) = 48 - 64 = -16$

abs max $(\sqrt{2}, 4)$

abs min $(2, -16)$

6. Sketch the graph of a function $f(x)$ so that all conditions below are satisfied. Be sure your graph is big enough so I can see it and it is properly labeled.

a) $\lim_{x\to\infty} f(x) = 3$, $\lim_{x\to -2^+} f(x) = \infty$, and $f(x)$ is defined for all x. (every x has a point)

HA $y = 3$ on right

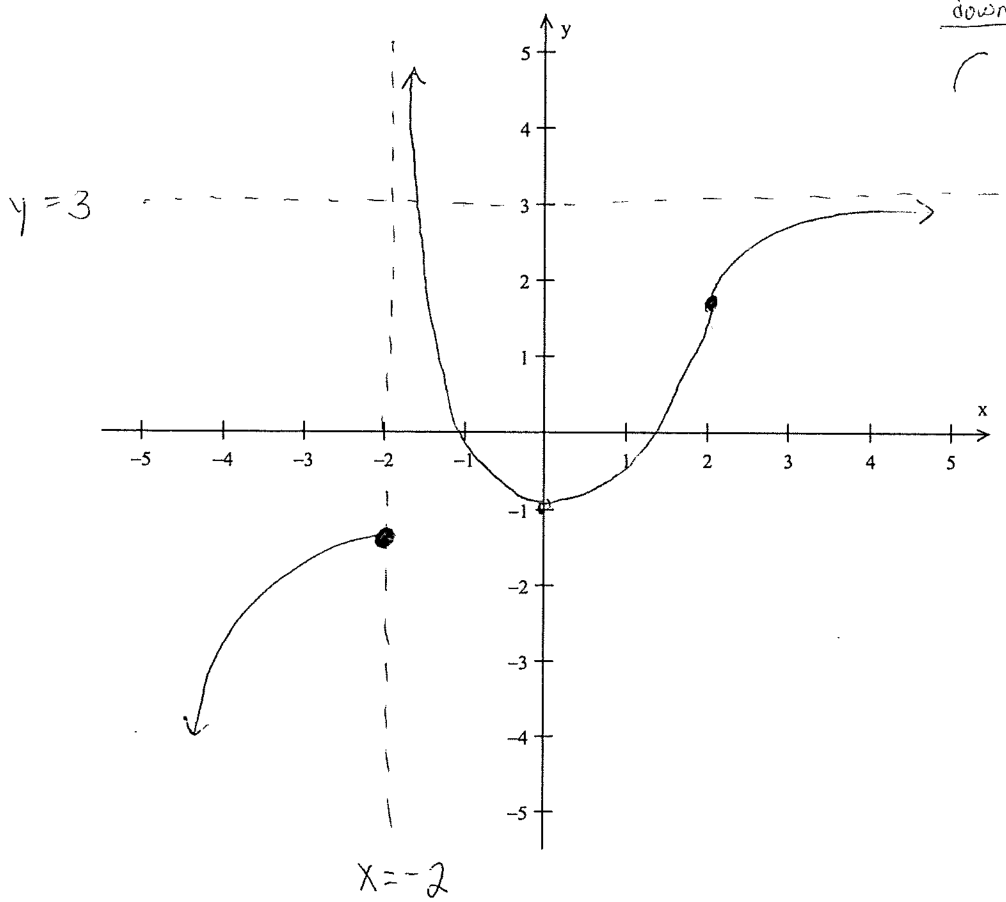
VA $x = -2$ on right

b) $f'(x) \overset{\text{dec}}{<} 0$ when $-2 < x < 0$, and $f'(x) \overset{\text{inc}}{>} 0$ when $x < -2$ and when $x > 0$.

c) $f''(x) \overset{\text{down}}{<} 0$ when $x < -2$ and when $x > 2$, but $f''(x) \overset{\text{up}}{>} 0$ when $-2 < x < 2$.

inc down | dec up | inc up | inc down — combo

-2 0 2



7. Find y' if $x^3 + (4xy^2) - 27 = y^4$.

$$3x^2 + 4y^2 + (4x)(2yy') = 4y^3y'$$

$$3x^2 + 4y^2 + 8xyy' = 4y^3y'$$

$$8xyy' - 4y^3y' = -3x^2 - 4y^2$$

$$y'(8xy - 4y^3) = -3x^2 - 4y^2$$

$$y' = \frac{-3x^2 - 4y^2}{8xy - 4y^3} = \frac{3x^2 + 4y^2}{4y^3 - 8xy}$$

either is ok

8. Suppose the demand equation for a product is $p = 400 - 2q$ and the average cost function is $\bar{c} = 0.2q + 4 + \dfrac{400}{q}$ where q is the number of units, p is the price per unit in dollars, and $\bar{c}$ is in dollars per unit. Find the maximum profit.

Profit = Rev − cost

$= p \cdot q - \bar{c} \cdot q$

$P = 400q - 2q^2 - (0.2q^2 + 4q + 400)$

$P = -2.2q^2 + 396q - 400$

$P' = -4.4q + 396 = 0$

$396 = 4.4q$

CN: $q = \dfrac{396}{4.4} = 90$ max or min?

1st deriv test: $\underset{90}{+ \;|\; -}$ P' max

OR 2nd deriv test: $P'' = -4.4$
$P''(90) = -4.4 < 0$ $\frown$ max

max profit occurs when $q = 90$, and that profit is...

$P(90) = -2.2(8100) + 396(90) - 400 = -17820 + 35640 - 400$

$= \$18220$