NAME KEY

Math 12
Test 2
Fall 2013

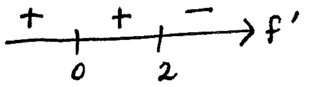
You have 50 minutes to complete this test. You must *show all work* to receive full credit. Work any 7 of the following 8 problems. Clearly **CROSS OUT** the problem you do not wish me to grade. Each problem is worth 14 points, and you get 2 points for free, for a total of 100 points. The answers will be posted on the electronic reserves later today.

1. Suppose that $f(x) = -3x^4 + 8x^3 - 10$. Find the intervals where $f(x)$ is increasing and where it is decreasing, and the intervals where it is concave up and where it is concave down (interval notation, please).

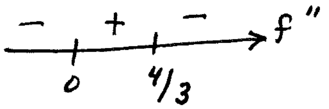
$f'(x) = -12x^3 + 24x^2$
$= -12x^2(x-2)$

$f''(x) = -36x^2 + 48x$
$= -12x(3x - 4)$

CN : $x = 0, 2$



IN : $x = 0, 4/3$



Results

inc on $(-\infty, 0) \cup (0, 2)$
dec on $(2, \infty)$
conc up on $(0, 4/3)$
conc down on
$(-\infty, 0) \cup (4/3, 0)$

2. For the following functions, find all horizontal and vertical asymptotes (remember that an asymptote is a LINE, not a number). If there are no asymptotes, say so.

a) $f(x) = 3 + \frac{x}{x^2+1} = \frac{3x^2 + 3 + x}{x^2+1}$

VA : none
HA : $y = 3$

b) $f(x) = 3x^3 + 2x - 5$

VA : none
HA : none

c) $f(x) = \frac{2x^2 + 7x - 4}{x^2 + x - 12} = \frac{(2x-1)(x+4)}{(x-3)(x+4)}$

VA : $x = 3$
HA : $y = 2$

3. Suppose that at price p, demand for a certain product is given by $q(p) = p^2 - 40p + 400$ when price is a positive value.

a) Find the price elasticity of demand when price is \$15. Is demand elastic or inelastic at this price?

$$E(p) = \frac{p}{q} \cdot \frac{q'}{1} = \frac{p}{p^2-40p+400} \cdot (2p-40)$$

$$E(15) = \frac{15}{225-600+400}(30-40) = \frac{-150}{225-600+400} = \frac{-150}{25} = -6 = \text{elasticity}$$

$|E(15)| = |-6| = 6 > 0$, so demand is elastic.

b) Complete this statement:

According to my answer in (a), if the price of \$15 goes *[✓ up]/down* (circle one)

by 1%, the demand for the product will go *up/[✓ down]* (circle one) by 6%.

c) Give an example of a product in the correct price range that might behave as described in (a) and (b).

luxury item, \$15, ... maybe a nice lunch.

4. For each of the following functions, find $f'(x)$. Do NOT simplify.

a) $f(x) = \sqrt[3]{\frac{8x^2-3}{x^2+2}} = \left(\frac{8x^2-3}{x^2+2}\right)^{1/3}$

$$f'(x) = \frac{1}{3}\left(\frac{8x^2-3}{x^2+2}\right)^{-2/3}\left(\frac{(16x)(x^2+2)-(8x^2-3)(2x)}{(x^2+2)^2}\right)$$

b) $f(x) = (x^2+1)^{100}\left(3x^{-4} + \frac{5}{x}\right)$

$$f'(x) = 100(x^2+1)^{99}(2x)(3x^{-4}+5x^{-1})$$
$$+ (x^2+1)^{100}(-12x^{-5}-5x^{-2})$$

5. Find all absolute maximum and minimum points of $f(x)=\frac{1-x}{x^2}$ on the interval $[1,4]$.

$$f'(x)=\frac{(-1)(x^2)-(1-x)(2x)}{x^4}=\frac{-x^2-2x+2x^2}{x^4}=\frac{x^2-2x}{x^4}=\frac{x-2}{x^3}$$

$\underline{CN}$: $x=0,\ 2$ ← not in $[1,4]$

$f(2)=-1/4$ absolute max $(1,0)$

$f(1)=0$ absolute min $(2,-1/4)$

$f(4)=-3/16$

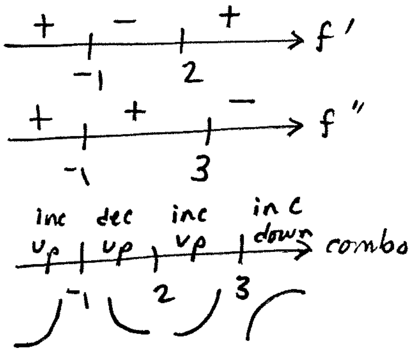
6. Sketch the graph of a function $f(x)$ so that all conditions below are satisfied. Be sure your graph is big enough so I can see it and it is properly labeled.

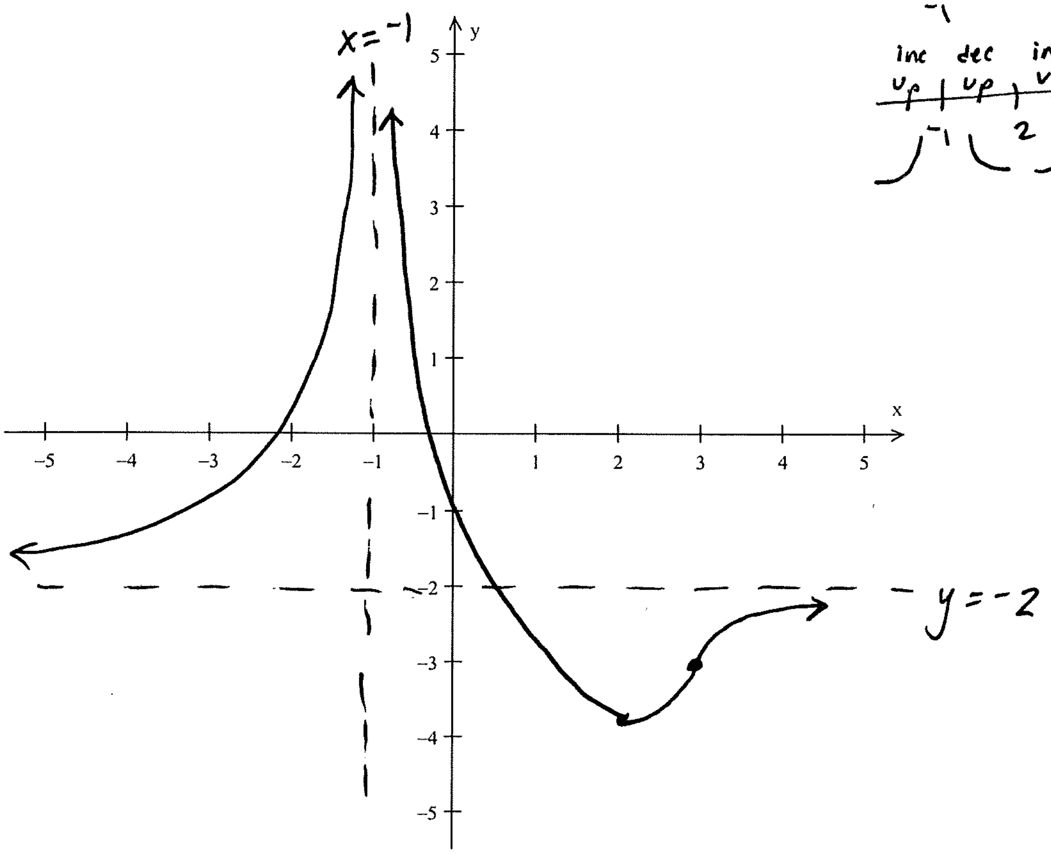
a) $\lim_{x\to\infty} f(x)=\lim_{x\to-\infty} f(x)=-2$. HA $y=-2$, both ends

b) $\lim_{x\to-1} f(x)=\infty$. VA $x=-1$, up on both sides

c) $f'(x)>0$ when $x<-1$ and when $x>2$.

d) $f''(x)>0$ when $x<-1$ and when $-1<x<3$.





7. Find y' if $2x^2y^2 - 4\sqrt{3x+2y} = 10$.

$$4x^2y^2 + 2x^2(2yy') - 2(3x+2y)^{-1/2}(3+2y') = 0$$

$$4x^2y^2 + 4x^2yy' - 6(3x+2y)^{-1/2} - 4y'(3x+2y)^{-1/2} = 0$$

$$4x^2yy' - 4y'(3x+2y)^{-1/2} = 6(3x+2y)^{-1/2} - 4x^2y^2$$

$$y'\left(4x^2y - 4(3x+2y)^{-1/2}\right) = 6(3x+2y)^{-1/2} - 4x^2y^2$$

$$y' = \frac{6(3x+2y)^{-1/2} - 4x^2y^2}{4x^2y - 4(3x+2y)^{-1/2}}$$

8. A satellite TV company has 4800 subscribers in a certain area who purchase an add-on package. They each pay \$18 per month for the bonus channels. The company can get 150 more subscribers for each 50 cent decrease in the monthly fee. What rate will yield the maximum revenue? What is this maximum revenue?

Let x = # of 50¢ price decreases.

Revenue = price · quantity

$R = (18 - 0.5x)(4800 + 150x)$

$R = 86400 - 2400x + 2700x - 75x^2$

$R = 86400 + 300x - 75x^2$

$R' = 300 - 150x = 0$

<u>CN</u> : $x = 2$

$R'' = -150$

$R''(2) = -150 < 0$

max.

Revenue is maximized when price is \$17.

Maximum Revenue is $R = (17)(4800+300)$

$R = \$86700$