NAME KEY

Math 12
Test 3
Spring 2013

You have 50 minutes to complete this test. You must *show all work* to receive full credit. Work any 6 of the following 7 problems. Clearly **CROSS OUT** the problem you do not wish me to grade. Each problem is worth 16 points, and you get 4 points for free, for a total of 100 points. The answers will be posted on the electronic reserves tomorrow.

1. Solve $y' = 8x^3 e^{-2y}$ if $y = 0$ when $x = 1$.

$$\frac{dy}{dx} = 8x^3 e^{-2y}$$

$$e^{2y}\,dy = 8x^3\,dx$$

$$\int e^{2y}\,dy = \int 8x^3\,dx$$

$$\frac{1}{2}e^{2y} = 2x^4 + C$$

If $x = 1$, $y = 0$, so

$$\frac{1}{2}e^0 = 2 + C$$

$$\frac{1}{2} = 2 + C$$

$$C = \frac{1}{2} - 2 = -3/2$$

$$\frac{1}{2}e^{2y} = 2x^4 - \frac{3}{2}$$

$$e^{2y} = 4x^4 - 3$$

$$2y = \ln(4x^4 - 3)$$

$$y = \frac{1}{2}\ln(4x^4 - 3)$$

2. Find $f'(x)$ for the following functions. DO NOT simplify!

(a) $f(x) = \ln(8x+5)^2$

$$f'(x) = \frac{1}{(8x+5)^2} \cdot 2(8x+5)(8)$$

OR:

$$f(x) = 2\ln(8x+5)$$

$$f'(x) = \frac{2}{8x+5} \cdot 8$$

(b) $f(x) = \dfrac{e^x + e^{-x}}{x^2}$

$$f'(x) = \frac{(e^x - e^{-x})(x^2) - (e^x + e^{-x})(2x)}{x^4}$$

3. a) How long will it take for \$2000 to grow to \$5000 if the investment earns interest at an annual rate of 8% compounded continuously?

$B = Pe^{rt}$

$5000 = 2000\,e^{0.08t}$

$2.5 = e^{0.08t}$

$\ln 2.5 = 0.08t$

$t = \frac{\ln 2.5}{0.08} \approx 11.45$ years

b) What quarterly interest rate would be required in order to accomplish the same goal in the same amount of time?

$B = P\left(1+\frac{r}{k}\right)^{kt}$

$5000 = 2000\left(1+\frac{r}{4}\right)^{4(11.45)}$

$2.5 = \left(1+\frac{r}{4}\right)^{45.81}$

$\ln 2.5 = 45.81 \ln\left(1+\frac{r}{4}\right)$

$\frac{\ln 2.5}{45.81} = \ln\left(1+\frac{r}{4}\right)$

$e^{\frac{\ln 2.5}{45.81}} = 1+\frac{r}{4}$

$r \doteq \left(e^{\frac{\ln 2.5}{45.81}}-1\right)\cdot 4$

$r \approx 0.08081$

8.081%

4. a) Solve for x: $\log_3(2x+3) = 4 - \log_3(x+6)$.

$\log_3(2x+3) + \log_3(x+6) = 4$

$\log_3(2x^2+15x+18) = 4$

$2x^2+15x+18 = 3^4 = 81$

$2x^2+15x-63 = 0$

$(2x+21)(x-3) = 0$

$x = \cancel{-\frac{21}{2}}$, $x=3$

not in domain, can't do $\log_3\left(-\frac{21}{2}+6\right)$

negative

b) Solve for x: $6e^{1-x}+1 = 25$.

$6e^{1-x} = 24$

$e^{1-x} = 4$

$1-x = \ln 4$

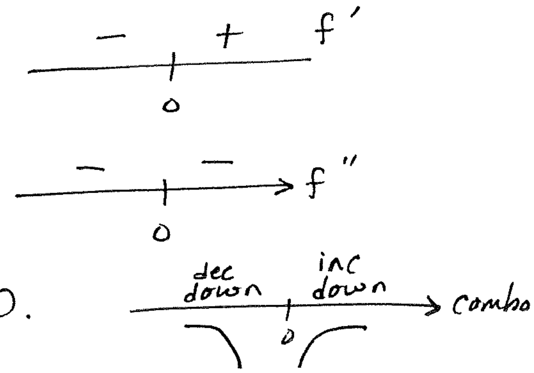
$1-\ln 4 = x$

$x \approx -0.386$

5. For the function $f(x) = 1 + \ln(x^2)$, list all intervals of increase and decrease, all maximum and minimum *points*, intervals where the function is concave up and concave down, all inflection *points*, and all asymptotes (or say there are none). Then sketch the graph of the function, being sure to label appropriately.

$$f'(x) = \frac{1}{x^2} \cdot 2x = \frac{2}{x} = 2x^{-1}$$

$$f''(x) = -2x^{-2} = \frac{-2}{x^2}$$



Notice $f(x)$ is defined for all $x \neq 0$.

VA : $x = 0$

HA : if $x \to \pm\infty$, y just gets bigger & bigger, none

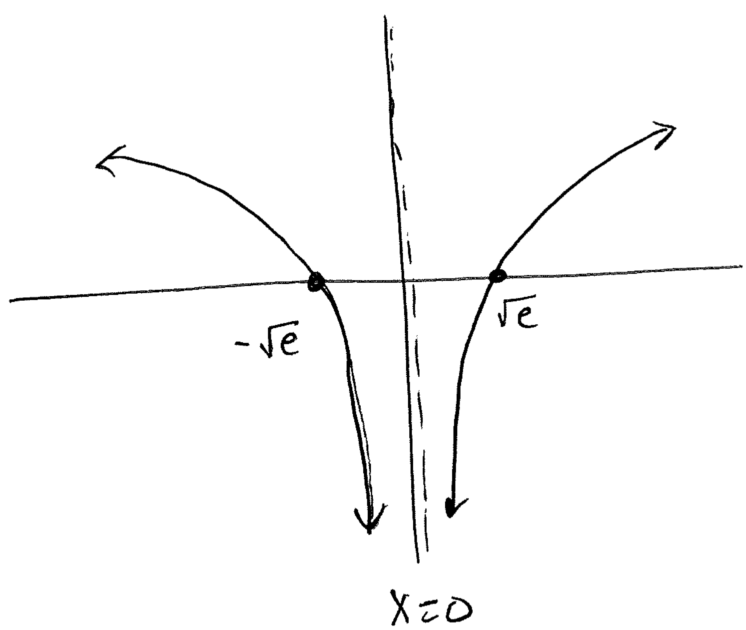
increasing on $(0, \infty)$

decreasing on $(-\infty, 0)$

concave up never

concave down on $(-\infty, 0) \cup (0, \infty)$

no max, min, or inflection points.



if $y = 0$,

$0 = 1 + \ln(x^2)$

$1 = \ln(x^2)$

$e = x^2$

$x = \pm\sqrt{e}$

x-intercepts

$(\pm\sqrt{e}, 0)$

not required, but nice to know

6. Evaluate the following integrals:

a) $\int x^3(x^2-1)^8 dx \quad = \frac{1}{2}\int x^2 u^8 du \quad = \frac{1}{2}\int (u+1)u^8 du$

Let $u = x^2 - 1$

then $du = 2x\,dx$, $\frac{1}{2}du = x\,dx$

notice $x^2 = u+1$

$$= \frac{1}{2}\int (u^9 + u^8)\,du$$

$$= \frac{1}{2}\left[\frac{1}{10}u^{10} + \frac{1}{9}u^9\right] + C$$

$$= \frac{1}{20}(x^2-1)^{10} + \frac{1}{18}(x^2-1)^9 + C$$

b) $\int\left(x^{\frac{2}{3}} - \frac{1}{x} + 5 + \sqrt{x}\right)dx \quad = \quad \int (x^{2/3} - x^{-1} + 5 + x^{1/2})\,dx$

$$= \frac{3}{5}x^{5/3} - \ln|x| + 5x + \frac{2}{3}x^{3/2} + C$$

7. Solve $\int xe^{-\frac{x}{2}} dx$

Let $u = x \qquad dv = e^{-\frac{1}{2}x}\,dx$

$du = dx \qquad v = \int e^{-\frac{1}{2}x}\,dx = -2e^{-\frac{1}{2}x}$

$$\int u\,dv = uv - \int v\,du$$

$$\int xe^{-\frac{1}{2}x}dx = -2xe^{-\frac{1}{2}x} - \int(-2e^{-\frac{1}{2}x})\,dx$$

$$= -2xe^{-\frac{1}{2}x} + 2\int e^{-\frac{1}{2}x}dx$$

$$= -2xe^{-\frac{1}{2}x} + 2(-2e^{-\frac{1}{2}x}) + C$$

$$= -2xe^{-\frac{1}{2}x} - 4e^{-\frac{1}{2}x} + C$$