NAME Key

Math 12
Test 3
Summer 2014

You have 60 minutes to complete this test. You must *show all work* to receive full credit. Work any 7 of the following 8 problems. Clearly **CROSS OUT** the problem you do not wish me to grade. Each problem is worth 14 points, and you get 2 points for free, for a total of 100 points. The answers will be posted on the electronic reserves later today.

1. Solve $\frac{dy}{dx} = \frac{1}{y(3x+1)}$.

$y\,dy = \frac{1}{3x+1}\,dx$

$\int y\,dy = \int \frac{1}{3x+1}\,dx$

$u = 3x+1$

$du = 3\,dx$

$\frac{1}{3}\,du = dx$

$\frac{1}{2}y^2 = \int \frac{1}{u}\left(\frac{1}{3}du\right)$

$\frac{1}{2}y^2 = \frac{1}{3}\int \frac{1}{u}\,du$

$\frac{1}{2}y^2 = \frac{1}{3}\ln|u| + C$

$\frac{1}{2}y^2 = \frac{1}{3}\ln|3x+1| + C$

$y^2 = \frac{2}{3}\ln|3x+1| + C$

$y = \pm\sqrt{\frac{2}{3}\ln|3x+1| + C}$

2. Evaluate $\int 5xe^{3x}\,dx$.

$u = 5x \qquad dv = e^{3x}dx$

$du = 5\,dx \qquad v = \int e^{3x}dx = \frac{1}{3}e^{3x}$

$\int u\,dv = uv - \int v\,du$

$= \frac{5}{3}xe^{3x} - \int \frac{1}{3}e^{3x}(5\,dx)$

$= \frac{5}{3}xe^{3x} - \frac{5}{3}\int e^{3x}dx$

$= \frac{5}{3}xe^{3x} - \frac{5}{9}e^{3x} + C$

3. Find all maxima, minima and inflection points of $f(x) = \ln(x^2+1)$. Also give the intervals where f is increasing, decreasing, concave up, and concave down. Then carefully sketch the graph of f.

$$f'(x) = \frac{2x}{x^2+1} = 0$$

crit #: $x = 0$

$$\xleftarrow{\quad - \quad}\underset{0}{|}\xrightarrow{\quad + \quad} f'$$

$$f''(x) = \frac{2(x^2+1) - (2x)(2x)}{(x^2+1)^2}$$

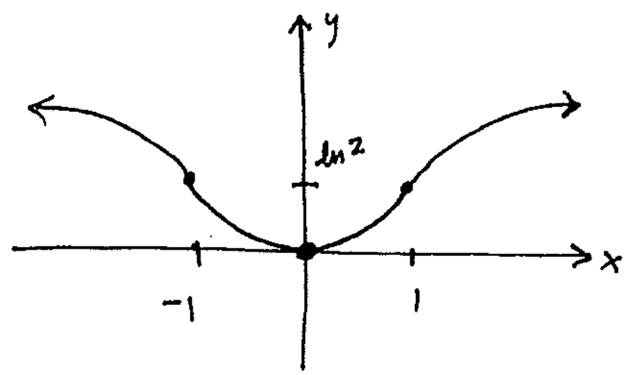
$$= \frac{2-2x^2}{(x^2+1)^2}$$

$$= \frac{2(1-x)(1+x)}{(x^2+1)^2}$$

possible inf. at: $x = \pm 1$

$$\xleftarrow{\quad - \quad}\underset{-1}{|}\xrightarrow{\quad + \quad}\underset{1}{|}\xrightarrow{\quad - \quad} f''$$

increasing on $(0, \infty)$
decreasing on $(-\infty, 0)$
min at $(0, 0)$
conc. up on $(-1, 1)$
conc down on $(-\infty, -1) \cup (1, \infty)$
inf pts $(\pm 1, \ln 2)$
$\approx (\pm 1, 0.69)$



4. Find $f'(x)$ for the following functions. DO NOT simplify!

(a) $f(x) = \dfrac{e^{-3x}}{x^2+1}$

$$f'(x) = \frac{(-3e^{-3x})(x^2+1) - (e^{-3x})(2x)}{(x^2+1)^2}$$

(b) $f(x) = x \ln\sqrt{x}$

$$f'(x) = (1)(\ln\sqrt{x}) + (x)\left(\frac{1}{\sqrt{x}}\right)\left(\frac{1}{2}x^{-1/2}\right)$$

5. Suppose you are offered two investment options. Option A offers a rate of return of 8.25% per year compounded quarterly. Option B offers a rate of 8.2% compounded continuously. Which option will give a better return on your investment? (Hint: Find the ending balance for a sample investment amount for one year).

As an example, if we invest \$100, we can calculate the ending balance for each option after 1 year:

Option A

$B = P\left(1+\frac{r}{k}\right)^{kt}$

$B = 100\left(1+\frac{.0825}{4}\right)^{4}$

$\approx \$108.51$

Option B

$B = Pe^{rt}$

$B = 100\,e^{0.082}$

$\approx \$108.55$

Option B gives a better return.

6. The rate at which a student employee can file papers is a function of the employee's experience. It is estimated that after t weeks on the job, the average student employee can file $Q(t) = 700 - 400e^{-0.5t}$ papers per hour.

a) How many papers can a new employee file per hour?

For a new employee, $t=0$, so

$Q(0) = 700 - 400e^{-0.5(0)} = 700 - 400 = 300$ papers per hour.

b) How many papers can a student employee with 6 weeks experience file per hour?

$Q(6) = 700 - 400e^{-0.5(6)} = 700 - 400e^{-3} \approx 680$ papers per hour

c) Approximately how many papers will a student employee be able to file per hour after an extended period of employment? (Show work and explain!)

Extended period of employment means $t \to \infty$.

$$\lim_{t\to\infty} Q(t) = \lim_{t\to\infty}\left(700 - 400e^{-0.5t}\right)$$

$$\approx 700 - 400\,e^{-(\text{big})}$$

$$\approx 700 - \frac{400}{\text{big \#}} \longrightarrow$$ denom gets big, so this term $\to 0$

$$\approx 700$$

7. a) If $\log_3(x-5)=2$, find x.

$$3^{\log_3(x-5)} = 3^2$$

$$x-5=9$$

$$x=14$$

b) If $\log_2 a=4$, $\log_2 b=3$, and $\log_2 c=6$, calculate $\log_2 \frac{a^3}{\sqrt{bc}}$.

$$\begin{aligned}\log_2 \frac{a^3}{(bc)^{1/2}} &= \log_2 a^3 - \log_2 (bc)^{1/2}\\ &= 3\log_2 a - \tfrac{1}{2}\log_2 (bc)\\ &= 3\log_2 a - \tfrac{1}{2}(\log_2 b + \log_2 c)\\ &= 3(4) - \tfrac{1}{2}(3+6)\\ &= 12 - 9/2\\ &= 15/2\end{aligned}$$

8. Evaluate the following integrals:

a) $\int (2x+6)^5\,dx \;=\; \int u^5 \left(\tfrac{1}{2}du\right) = \tfrac{1}{2}\int u^5\,du$

$u = 2x$

$du = 2dx$

$\tfrac{1}{2}du = dx$

$$= \tfrac{1}{2}\left(\frac{u^6}{6}\right) + C$$

$$= \frac{(2x+6)^6}{12} + C$$

b) $\int \left(3\sqrt{x} - \frac{2}{x^3} + \frac{1}{x}\right) dx$

$$= \int (3x^{1/2} - 2x^{-3} + x^{-1})\,dx$$

$$= \frac{3x^{3/2}}{3/2} - \frac{2x^{-2}}{-2} + \ln|x| + C = 2x^{3/2} + x^{-2} + \ln|x| + C$$