Chapter 2, Section 3

2. $f(x) = (x-5)(1-2x)$

$$f'(x) = (1)(1-2x) + (x-5)(-2)$$
$$= 1 - 2x - 2x + 10$$
$$= -4x + 11$$

4. $y = 400(15 - x^2)(3x-2)$

$$y' = 400\left[-2x(3x-2) + (15-x^2)(3)\right]$$
$$= 400\left[-6x^2 + 4x + 45 - 3x^2\right]$$
$$= -3600x^2 + 1600x + 18000$$

8. $y = \dfrac{2x-3}{5x+4}$

$$y' = \frac{2(5x+4) - (2x-3)(5)}{(5x+4)^2} = \frac{10x + 8 - 10x + 15}{(5x+4)^2} = \frac{23}{(5x+4)^2}$$

14. $f(x) = \dfrac{x^2+2x+1}{3}$

$$f'(x) = \frac{(2x+2)(3) - (x^2+2x+1)(0)}{9} = \frac{2x+2}{3}$$

other way: $f(x) = \frac{1}{3}x^2 + \frac{2}{3}x + \frac{1}{3}$, so $f'(x) = \frac{2}{3}x + \frac{2}{3}$.

16. $g(x) = \dfrac{(x^2+x+1)(4-x)}{2x-1}$

$$g'(x) = \frac{\left[(2x+1)(4-x) + (x^2+x+1)(-1)\right](2x-1) - (x^2+x+1)(4-x)(2)}{(2x-1)^2}$$

$$= \frac{(8x - 2x^2 + 4 - x - x^2 - x - 1)(2x-1) - (4x^2 - x^3 + 4x - x^2 + 4 - x)(2)}{(2x-1)^2}$$

$$= \frac{16x^2 - 8x - 4x^3 + 2x + 8x - 4 - 2x^2 + x - 2x^3 + x^2 - 2x^2 + x - 2x + 1 - 8x^2 + 2x^3 - 8x + 2x^2 - 8 + 2x}{(2x-1)^2}$$

$$= \frac{-2x^3 + 7x^2 - 4x - 11}{(2x-1)^2}$$

Chapter 2, Section 3

18. $y = (x^2+3x-1)(2-x)$. Find tangent line at $x=1$.

$y' = (2x+3)(2-x) + (x^2+3x-1)(-1)$

$m = y'(1) = (5)(1) + (3)(-1) = 2$ = slope

point: $x=1$, $y = (1+3-1)(2-1) = 3$ $(1,3)$

line: $y-3 = 2(x-1)$, or $y = 2x+1$

28. $y = \frac{5x+7}{2-3x}$. Find normal line at $(1,-12)$

$y' = \frac{(5)(2-3x)-(5x+7)(-3)}{(2-3x)^2}$

$y'(1) = \frac{(5)(-1)-(12)(-3)}{(2-3)^2} = -5+36 = 31 = m_{tan}$.

$m_{norm} = \frac{-1}{31}$ (perpendicular to tangent)

Normal Line: $y+12 = \frac{-1}{31}(x-1)$ or $y = \frac{-1}{31}x - \frac{371}{31}$

30. $y = \frac{2x-3}{x^3}$

a) $y' = \frac{2(x^3)-(2x-3)(3x^2)}{x^6} = \frac{2x^3-6x^3+9x^2}{x^6} = \frac{-4x+9}{x^4}$

b) $y = x^{-3}(2x-3)$

$y' = (-3x^{-4})(2x-3) + (x^{-3})(2) = -6x^{-3} + 9x^{-4} + 2x^{-3} = -4x^{-3} + 9x^{-4}$

c) $y = 2x^{-2} - 3x^{-3}$

$y' = -4x^{-3} + 9x^{-4}$

d) All give same result, of course.

Chapter 2, Section 3

36. $S(t) = 50\left(1 - \frac{t^2}{15}\right)^3$ pounds

a) originally, amt of sand was

$S(0) = 50(1-0)^3 = 50$ pounds

b) $S(t) = 50\left(1 - \frac{1}{5}t^2 + \frac{1}{75}t^4 - \frac{1}{3375}t^6\right)$

$S'(t) = 50\left(-\frac{2}{5}t + \frac{4}{75}t^3 - \frac{2}{1125}t^5\right)$

$S'(1) = 50\left(-\frac{2}{5} + \frac{4}{75} - \frac{2}{1125}\right) = 50\left(\frac{-450}{1125} + \frac{60}{1125} - \frac{2}{1125}\right) = \frac{-19600}{1125}$

$= -\frac{784}{45} \approx -17.42$ pounds/sec

c) When all sand is gone, $S(t)=0$, so

$50\left(1 - \frac{t^2}{15}\right)^3 = 0$

$1 = t^2/15$

$t = \sqrt{15}$ sec ≈ 3.9 sec

Rate of leakage when $t = \sqrt{15}$ is

$S'(\sqrt{15}) = 50\sqrt{15}\left(\frac{-2}{5} + \frac{60}{75} - \frac{2(225)}{1125}\right) = 50\sqrt{15}\left(\frac{-2}{5} + \frac{4}{5} - \frac{2}{5}\right)$

$= 50\sqrt{15}\,(0) = 0$ pounds/sec (of course, since nearly gone!)

42. a) $\frac{d}{dx}\left(\frac{fg}{h}\right) = \frac{(fg)'h - (fg)h'}{h^2} = \frac{(f'g + fg')h - fgh'}{h^2}$

$= (f'gh + fg'h - fgh')/h^2$

b) $y = \frac{(2x+7)(x^2+3)}{3x+5}$

$y' = \frac{(2)(x^2+3)(3x+5) + (2x+7)(2x)(3x+5) - (2x+7)(x^2+3)(3)}{(3x+5)^2}$

$= \frac{6x^3+10x^2+18x+30 + 12x^3+62x^2+70x - 6x^3 - 21x^2 - 18x - 63}{(3x+5)^2}$

$y' = \frac{12x^3+51x^2+70x-33}{(3x+5)^2}$

Chapter 2, Section 4

4. $C(x) = \frac{1}{4}x^2 + 3x + 67, \quad p(x) = \frac{1}{5}(45 - x) = 9 - \frac{1}{5}x.$

a) marginal cost $= C'(x) = \frac{1}{2}x + 3$

Revenue $= x \cdot p(x) = 9x - \frac{1}{5}x^2 = R(x)$

marginal revenue $= R'(x) = 9 - \frac{2}{5}x$

b) Cost of 4^{th} unit $\approx C'(3) = \frac{3}{2} + 3 = \frac{9}{2} = 4.5$

c) Actual Cost of 4^{th} unit $= C(4) - C(3) = (4 + 12 + 67) - (\frac{9}{4} + 9 + 67)$

$= 7 - \frac{9}{4} = \frac{19}{4} = 4.75$

d) Revenue from 4^{th} unit $\approx R'(3) = 9 - \frac{6}{5} = \frac{39}{5} = 7.8$

e) Actual Revenue from 4^{th} unit $= R(4) - R(3) = (36 - \frac{16}{5}) - (27 - \frac{9}{5})$

$= 9 - \frac{7}{5} = \frac{38}{5} = 7.6$

8. $C(x) = \frac{2}{7}x^2 + 65, \quad p(x) = \frac{12 + 2x}{3 + x}$

a) marginal cost $= C'(x) = \frac{4}{7}x$

revenue $= R(x) = \frac{12x + 2x^2}{3 + x}$

marginal revenue $= R'(x) = \frac{(12 + 4x)(3 + x) - (12x + 2x^2)(1)}{(3 + x)^2} = \frac{36 + 24x + 4x^2 - 12x - 2x^2}{(3 + x)^2}$

$= \frac{2x^2 + 12x + 36}{(3 + x)^2} = \frac{2(x^2 + 6x + 18)}{(3 + x)^2}$

b) Cost of 4^{th} unit $\approx C'(3) = \frac{12}{7} \approx 1.714$

c) Actual cost of 4^{th} unit $= C(4) - C(3) = (\frac{32}{7} + 65) - (\frac{18}{7} + 65) = 2$

d) Revenue from 4^{th} unit $\approx R'(3) = \frac{2(9 + 18 + 18)}{36} = 2.5$

e) Actual revenue from 4^{th} unit $= R(4) - R(3) = \frac{48 + 32}{7} - \frac{36 + 18}{6} = \frac{17}{7} \approx 2.43$

9. $f(x) = x^2 - 3x + 5 \qquad \Delta f \approx f'(x_0)\Delta x$, where $x_0 = 5$, $\Delta x = .3$

$f'(x) = 2x - 3 \qquad \Delta f \approx 7(.3) = 2.1$

$f(5.3) = f(5) + \Delta f$

$\approx f(5) + f'(x_0)\Delta x = (25 - 15 + 5) + 2.1$

≈ 17.1

Chapter 2, Section 4

12. Find % change in $f(x) = 3x + \frac{2}{x}$ when x decreases from 5 to 4.6. (estimate only)

$$\%\text{ change} = 100\frac{\Delta f}{f(x)} \approx 100\frac{f'(x)\Delta x}{f(x)}$$

use $x = 5$, $\Delta x = -0.4$, $f'(x) = 3 - 2x^{-2}$.

$$\%\text{ change} \approx \frac{100(3-2(5)^{-2})(-0.4)}{3(5)+2/5} = \frac{100(3-\frac{2}{25})(-0.4)}{77/5}$$

$$\approx \frac{-40(73/25)}{77/5} = -\frac{584}{77} = -7.58\%$$

16. $C(q) = \frac{1}{6}q^3 + 642q + 400$ dollars. Currently $q = 4$. Estimate amount by which production should be decreased to make cost go down \$130.

Want $\Delta C = -130$. $\Delta C \approx C'(q)\Delta q$.

$C'(q) = \frac{1}{2}q^2 + 642$.

When $q = 4$, $\Delta C \approx C'(4)\Delta q$

$$-130 \approx \left(\tfrac{1}{2}(16) + 642\right)\Delta q = 650\,\Delta q$$

$$\frac{-13}{65} \approx \Delta q.$$

Decrease production by $13/65$ of a unit to make $C \downarrow$ \$130.

18. $A(t) = 0.1t^2 + 10t + 20$ thousand dollars t years after 1996. Estimate the % change in earnings in 3rd quarter of 2000.

$$\%\text{ change} = 100\frac{\Delta A}{A(t)} \approx 100\frac{A'(t)\Delta t}{A(t)}.$$

For 3rd quarter of 2000, we go from $t = 4.5$ to $t = 4.75$. Use $t = 4.5$, $\Delta t = 0.25$, so $A'(t) = 0.2t + 10$.

$$\%\text{ change} \approx 100\frac{A'(4.5)\cdot(0.25)}{A(4.5)} \approx 100\frac{(10.9)(0.25)}{2.025+45+20} \approx \frac{272.5}{67.025} \approx 4.07\%$$

Chapter 2, Section 4

24. $C(t) = 100t^2 + 400t + 5000$ in t years from now. Estimate C 6 months from now.

$C(0) = 5000$ (now)

$C(t+\Delta t) = C(t) + \Delta C$

$\approx C(t) + C'(t)\Delta t$

$C'(t) = 200t + 400$

Use $t=0$, $\Delta t = 0.5$ years, so $C'(0) = 400$

$C(0.5) \approx C(0) + C'(0)(0.5)$

$\approx 5000 + 400(0.5)$

≈ 5200

29. Output $Q = 600K^{1/2}L^{1/3}$. Estimate % increase in output if K is constant and L increases by 2%

$Q'(L) = 200K^{1/2}L^{-2/3}$

$\Delta L = 0.02L$

$$\% \text{ change} \approx 100 \frac{Q'(L)\Delta L}{Q(L)}$$

$$\approx \frac{100(200K^{1/2}L^{-2/3})(0.02L)}{600K^{1/2}L^{1/3}}$$

$$\approx \frac{400K^{1/2}L^{1/3}}{600K^{1/2}L^{1/3}}$$

$$\approx \frac{2}{3}\%$$