Chapter 3, Section 4

4. $f(x) = x^5 - 5x^4 + 1$ on $0 \le x \le 5$.

$f'(x) = 5x^4 - 20x^3 = 5x^3(x-4)$ Test CN'S (0 and 4) and endpts (0 and 5)

$f(0) = 1$

$f(4) = 1024 - 1280 + 1 = -255$ abs max at $(0,1)$ and $(5,1)$

$f(5) = 3125 - 3125 + 1 = 1$ abs min at $(4, -255)$

8. $f(t) = \dfrac{t^2}{t-1}$ $\quad -2 \le t \le -1/2$

$f'(t) = \dfrac{2t(t-1) - t^2}{(t-1)^2} = \dfrac{t^2 - 2t}{(t-1)^2} = \dfrac{t(t-2)}{(t-1)^2}$ CN'S: 0, 2, 1 not in interval! only need to test endpoints.

$f(-2) = \dfrac{4}{-2-1} = -4/3$

$f(-1/2) = \dfrac{1/4}{-1/2 - 1} = \dfrac{1/4}{-3/2} = \dfrac{1}{4} \cdot \dfrac{-2}{3} = -\dfrac{1}{6}$

abs max $\left(-\frac{1}{2}, -\frac{1}{6}\right)$, abs min $\left(-2, -\frac{4}{3}\right)$

16. $f(x) = \dfrac{1}{(x+1)^2} = (x+1)^{-2}$ for $x \ge 0$

$f'(x) = -2(x+1)^{-3}$ CN: $x = -1$, not in interval, so test endpoint:

$f(0) = 1$. This is either the smallest or largest, since it's the only endpt and there are no CN's in interval.

Notice for $x \ge 0$, $f'(x) = -2(x+1)^3 < 0$. This means f is decreasing for all $x \ge 0$, so $(0,1)$ is the absolute max.

Chapter 3, Section 4

18. $p(q) = 37 - 2q$

$C(q) = 3q^2 + 5q + 75$

$R(q) = p(q) \cdot q = 37q - 2q^2$

a) $P(q) = R(q) - C(q) = 37q - 2q^2 - 3q^2 - 5q - 75 = -5q^2 + 32q - 75$

$\left.\begin{aligned} R'(q) &= 37 - 4q \\ C'(q) &= 6q + 5 \end{aligned}\right\}$ to maximize $P(q)$, we need $R'(q) = C'(q)$, so

$37 - 4q = 6q + 5$

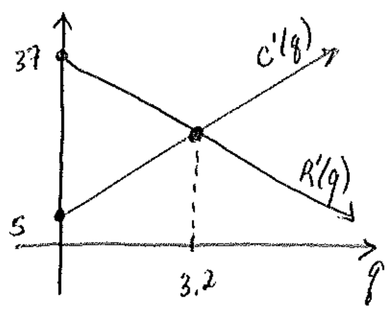
$32 = 10q$, so $q = 3.2$.

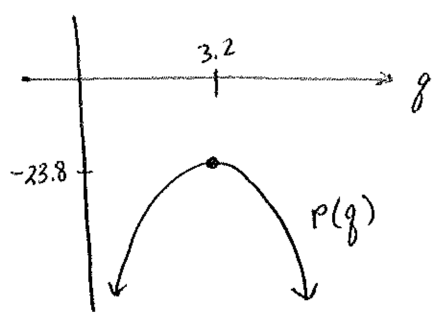
we also need $R''(q) < C''(q)$.

$\underset{-4}{R''(q)} < \underset{6}{C''(q)}$ so yes, this is true.

$q = 3.2$ will maximize profit.

$P(3.2) = -23.8 =$ maximum profit





b) $A(q) = \dfrac{C(q)}{q} = 3q + 5 + 75/q$

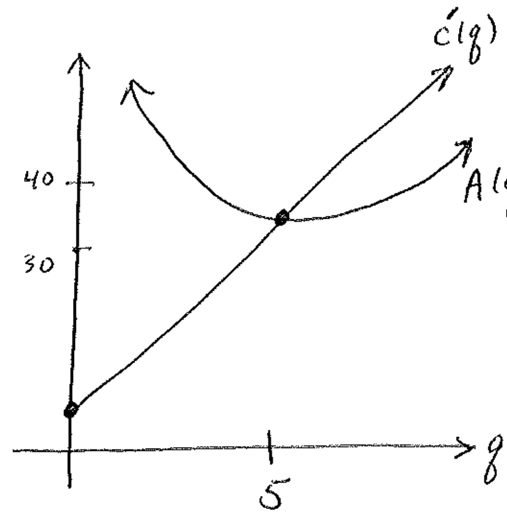
$A(q)$ is minimized when $A(q) = C'(q)$, so

$3q + 5 + 75/q = 6q + 5$

$75/q = 3q$

$25 = q^2$

$\boxed{q = 5}$



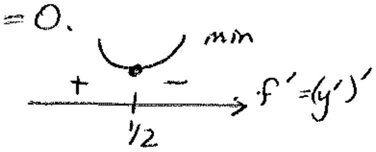
Chapter 3, Section 4

24. $y = 2x^3 - 3x^2 + 6x$

slope $= y' = 6x^2 - 6x + 6$

To minimize slope, find slope$' = y'' = 0$.

$y'' = 12x - 6 = 0$, CN: $x = 1/2$



The slope when $x = 1/2$ is $y'(\frac{1}{2})$.

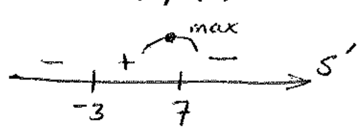
$m = 6(\frac{1}{4}) - 6(\frac{1}{2}) + 6 = \frac{3}{2} - 3 + 6 = 9/2$

32. $S(x) = \frac{1}{29}(-x^3 + 6x^2 + 63x + 1080)$ for $0 \le x \le 12$.

She should announce her candidacy at a time that will give her the greatest support, so maximize S.

$S'(x) = \frac{1}{29}(-3x^2 + 12x + 63) = -3/29\,(x^2 - 4x - 21)$

$= \frac{-3}{29}(x-7)(x+3)$



Notice $x = -3$ is not in the interval $[0, 12]$. Check CN's and endpoints:

$S(0) = \frac{1080}{29} \approx 37.24\%$

$S(7) = \frac{1472}{29} \approx 51.1\%$ ←

$S(12) = \frac{972}{29} \approx 33.51\%$

She will have the maximum amount of support if she announces her candidacy 7 months prior to the election. If the election is held in April , she should announce her candidacy in April. At the time of the election, she should win with 51.1% of the vote.

Chapter 3, Section 4

34. x = total national income, $C(x)$ = total national consumption.
$C'(x)$ = marginal propensity to consume
$S = x - C$ is total national savings
$S'(x)$ = marginal propensity to save

$C(x) = 8 - 0.8x - 0.8\sqrt{x}$
marginal propensity to consume is $C'(x) = -0.8 - 0.4x^{-1/2}$

Find the value of x that gives the smallest total savings
$S(x) = x - C(x)$. Minimize this, so
$S'(x) = 1 - C'(x) = 1 + 0.8 + 0.4x^{-1/2} = 0$
$0.4x^{-1/2} = -1.8$
$-\frac{0.4}{1.8} = \sqrt{x}$ But this can't be negative, so NO critical points.
Only need to check the endpoints. Notice $x=0$ is the only boundary x-value, and $S'(x) > 0$ for all x. Savings is always increasing, so $x=0$ gives the smallest savings, which will be
$S(0) = 0 - C(0) = 0 - (8 - 0 - 0) = -8$

Chapter 3, Section 5

6. Cost is \$3 per book. Let x = # \$1 reductions in price.

price $= 15 - x$. Number sold $= 200 + 20x$

Maximize Profit. $P(x) = R(x) - C(x)$

$P(x) = (15 - x)(200 + 20x) - 3(200 + 20x)$

$= -20x^2 + 40x + 2400$

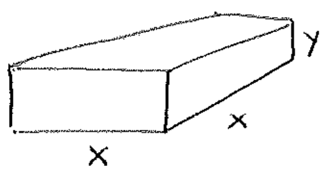
$P'(x) = -40x + 40 = 0$ $\quad$ CN: $x = 1$

$P''(x) = -40$

$P''(1) = -40 < 0$ $\quad$ $x = +1$ is maximize profit.

The price that will maximize profit is \$14.

15.



Cost of bottom $= 4x^2$

Cost of one side $= 3xy$

Total cost $= 4x^2 + 4(3xy) = 48$

$y = \frac{48 - 4x^2}{12x} = \frac{4}{x} - \frac{1}{3}x$

Maximize volume.

$V = x^2 y = x^2\left(\frac{4}{x} - \frac{1}{3}x\right) = 4x - \frac{1}{3}x^3$

$V' = 4 - x^2 = 0$ $\quad$ CN: $x = 2$, $\cancel{x = -2}$

$V'' = -2x$

$V''(2) = -4 < 0$ $\quad$ $x = 2$ gives maximum volume.

$y = \frac{4}{2} - \frac{1}{3}(2) = \frac{4}{3}$ $\quad$ so dimensions for maximum volume are $2 \times 2 \times \frac{4}{3}$

Chapter 3, Section 5

27. cost of production = setup cost + operating cost

Let x = # machines used

setup cost = $20x$

operating cost = 15 (# hours).

To make 8000 units... $8000 = x\text{ machines} \times \dfrac{\text{\# hours}}{\text{machine}} \times \dfrac{30 \text{ kickboards}}{1 \text{ hour}}$

$$\frac{8000}{30x} = \text{\# hours}$$

$$C(x) = 20x + 15\left(\frac{8000}{30x}\right) = 20x + 4000x^{-1}$$

a) minimize $C(x)$: $C'(x) = 20 - 4000x^{-2} = 0$

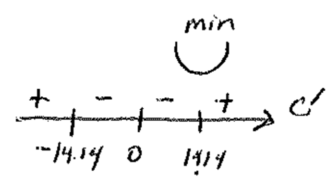
$4000 = 20x^2$

$200 = x^2$

$x = \pm\sqrt{200} \approx \pm 14.14$

also, C' undef when $x=0$

$x \doteq 14.14$ will minimize C.



But there are only 10 machines! Since C decreases between $x=0$ and $x=14.14$, $x=10$ will give minimum cost.

b) Supervisor will earn $\$15/\text{hour} \cdot \text{\# hours}$

Using 10 machines, # hours is $\dfrac{8000}{30(10)} = \dfrac{80}{3}$ hours

money earned = $15(80/3) = \$400$

c) Setup cost for 10 machines is $\$20(10) = \$200$.

Chapter 3, Section 5

28. Minimize total cost = storage cost + ordering cost

Let x = # cases per order. Then # orders $= 600 \text{ cases} \times \frac{1 \text{ order}}{x \text{ cases}} = \frac{600}{x}$

$C(x) = (\#\text{ in storage})(\text{storage fee}) + (\#\text{orders})(\text{cost per order})$

$= \left(\frac{x}{2}\right)(0.90) + \left(\frac{600}{x}\right)(30)$

$C(x) = 0.45x + 18000x^{-1}$

$C'(x) = 0.45 - 18000x^{-2} = 0 \longrightarrow \frac{18000}{0.45} = x^2$

$x = 200, \quad \cancel{x = -200}$

$C''(x) = 36000x^{-3}, \quad C''(200) = \frac{36000}{(200)^3} > 0$, so $x = 200$ minimizes C.

The firm should order 200 cases at a time.

32. $x(p) = 500 - 2p$ units demanded. p = price, and $0 \le p \le 250$.

a) $E(p) = \frac{p}{x} \cdot x' = \frac{p}{500-2p}(-2) = \frac{-2p}{500-2p}$

$|E(p)| = 1$ when either

① $1 = \frac{-2p}{500-2p}$ OR ② $-1 = \frac{-2p}{500-2p}$

$500 - 2p = -2p$ $\qquad$ $-500 + 2p = -2p$

$500 = 0$, no solutions $\qquad$ $4p = 500$, $p = 125$

inelastic | elastic | $E(p)$

0 $\quad <1 \quad$ 125 $\quad >1 \quad$ 250

So demand is inelastic if $0 \le p < 125$, unitary if $p = 125$, elastic if $125 < p \le 250$.

Another way: $|E(p)| < 1$ means $-1 < E(p) < 1$, so $-1 < \frac{-2p}{500-2p} < 1$.

Since $0 \le p \le 250$, $500 - 2p$ is nonnegative, so we can multiply by it, so $-500 + 2p < -2p < 500 - 2p$

$-500 < 4p < 0 < 500$ $\qquad$ $p < 125$ makes $|E(p)| < 1$.

Chapter 3, Section 5

32. (cont)

b) From the analysis on pg 273,

demand is elastic $\longrightarrow$ revenue is decreasing $(125, 250)$

demand is inelastic $\longrightarrow$ revenue is increasing $(0, 125)$

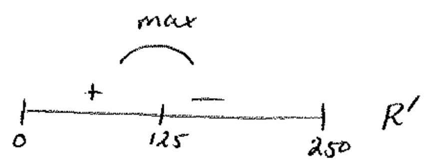
$R(p)$ increases on $(0,125)$ and decreases on $(125, 250)$, so

Revenue is maximized when $p = 125$.

c) $R(p) = xp = 500p - 2p^2$

$R'(p) = 500 - 4p = 0$

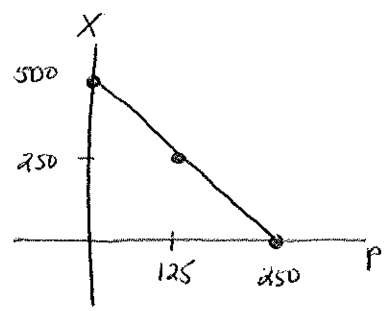
CN: $p = 125$



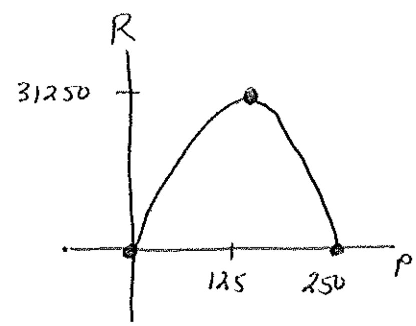
Revenue is maximized when $p = 125$.

d) Graph $x(p)$ and $R(p)$

p	$x(p)$	$R(p)$
0	500	0
125	250	31250
250	0	0



demand
$x(p)$



revenue
$R(p)$

Chapter 3, Section 5

39. $x = \#$ years from now

$V(x) =$ value at time x

$\frac{dV}{dx} = 53 - 10x$ dollars change in value per year

Storage is \$3 per case per year

Want to maximize profit per case.

Profit per case $= P(x) = V(x) - 3x$

$P'(x) = \frac{dV}{dx} - 3 = 53 - 10x - 3$

$P'(x) = 50 - 10x$ $\qquad \underset{5 \atop \max}{+ \mid -} \; P'$

Profit is maximized 5 years from now.

40. $x =$ speed in mph. $40 \le x \le 65$. Truck gets $\frac{480}{x}$ miles per gallon. Diesel costs \$1.12/gal, driver is paid \$12 per hour. Find the speed that minimizes cost.

We will minimize cost per hour.

$C(x) =$ driver pay + fuel cost, all in dollars per hour.

$= \$12/\text{hour} + \left(\frac{1.12}{\text{gal}}\right)\left(\frac{x}{480}\frac{\text{gal}}{\text{mile}}\right)\left(\frac{x \text{ miles}}{1 \text{ hour}}\right)$

$= 12 + \frac{1.12x^2}{480}$ dollars per hour

$= 12 + \frac{7}{3000}x^2$

$C'(x) = \frac{7}{1500}x$

<u>CN</u>: $x = 0$ $\qquad \underset{0}{- \mid +} \; C'$

Since cost increases for all speeds x greater than 0, cost is minimized at the lowest speed allowed, 45 mph.

Chapter 3, Section 5

53. Total cost $C(x) = \frac{7}{8}x^2 + 5x + 100$ dollars
price $p(x) = 15 - \frac{3}{8}x$ when x units are produced.
tax of t dollars per unit.

a) Profit $= P(x) = 15x - \frac{3}{8}x^2 - \frac{7}{8}x^2 - 5x - 100 - tx$

$$= \frac{-5}{4}x^2 + 10x - 100 - tx$$

$$P'(x) = \frac{-5}{2}x + 10 - t = 0 \quad \text{(remember } t \text{ is constant)}$$

$$x = -\frac{2}{5}(t - 10) = \frac{2}{5}(10 - t)$$

$$\underset{\frac{2}{5}(10-t)}{+ \quad | \quad -} \longrightarrow P'$$

fill in -10000 on left
10000 on right
just to be safe.

b) To maximize tax revenue, we know monopolist will maximize profit, so $\frac{2}{5}(10 - t)$ units will be produced. $x = \frac{2}{5}(10 - t)$.

Tax collected $= T(t) = tx = t\left(\frac{2}{5}\right)(10 - t)$

$$T(t) = -\frac{2}{5}t^2 + 4t$$

$$T'(t) = -\frac{4}{5}t + 4 = 0$$

$$\underset{5}{+ \quad | \quad -} \longrightarrow T'$$

Maximum tax revenue when $t = 5$

c) Each unit is taxed \$5.
With no tax, $t=0$. To maximize profit, monopolist produces $\frac{2}{5}(10 - 0) = 4$ units, and price is $15 - \frac{3}{8}(4) = \$13.50$.
With a \$5 tax, $t = 5$, and $\frac{2}{5}(10 - 5) = 2$ units are produced. price is $15 - \frac{3}{8}(2) = \$14.25$. The price is 75¢ higher when the tax is imposed. The consumer pays 75¢ of the tax, and the monopolist pays \$4.25 of the tax.