Section 5.2 – Integration by Substitution

2. $\int e^{5x}\,dx$. $\quad u = 5x$, $du = 5\,dx$, $\frac{1}{5}du = dx$

$$\int e^{5x}dx = \int e^{u}\left(\tfrac{1}{5}du\right) = \tfrac{1}{5}\int e^{u}du = \tfrac{1}{5}e^{u}+C = \tfrac{1}{5}e^{5x}+C.$$

4. $\int \frac{1}{3x+5}\,dx$. $\quad u = 3x+5$, $du = 3\,dx$, $\frac{1}{3}du = dx$

$$\int \frac{1}{3x+5}dx = \int \frac{1}{u}\left(\tfrac{1}{3}du\right) = \tfrac{1}{3}\int \tfrac{1}{u}du = \tfrac{1}{3}\ln|u|+C = \tfrac{1}{3}\ln|3x+5|+C.$$

6. $\int [(x-1)^5 + 3(x-1)^2 + 5]\,dx$ $\quad u = x-1$, $du = dx$

$$= \int (u^5 + 3u^2 + 5)\,du = \tfrac{1}{6}u^6 + u^3 + 5u + C$$
$$= \tfrac{1}{6}(x-1)^6 + (x-1)^3 + 5(x-1) + C$$

10. $\int 3t\sqrt{t^2+8}\,dt$ $\quad u = t^2+8$, $du = 2t\,dt$, $\frac{3}{2}du = 3t\,dt$

$$\int 3t\sqrt{t^2+8}\,dt = \int u^{1/2}\left(\tfrac{3}{2}du\right) = \tfrac{3}{2}\int u^{1/2}du = \tfrac{3}{2}\left(\tfrac{2}{3}u^{3/2}\right)+C = (t^2+8)^{3/2}+C$$

12. $\int x^5 e^{1-x^6}\,dx = \int e^{u}\left(\frac{-1}{6}du\right) = -\frac{1}{6}\int e^{u}du$

$u = 1-x^6$, $du = -6x^5\,dx$, $-\frac{1}{6}du = x^5\,dx$

$$= \frac{-1}{6}e^{u}+C = \frac{-1}{6}e^{1-x^6}+C$$

14. $\int \frac{y^2}{(y^3+5)^2}\,dy = \int \frac{\frac{1}{3}\,du}{u^2}$

$u = y^3+5$, $du = 3y^2\,dy$, $\frac{1}{3}du = y^2\,dy$

$$= \tfrac{1}{3}\int u^{-2}du = \tfrac{1}{3}\left(\frac{u^{-1}}{-1}\right)+C = -\tfrac{1}{3}(y^3+5)^{-1}+C = \frac{-1}{3(y^3+5)}+C$$

Section 5.2 - cont

18. $\displaystyle\int \frac{10x^3-5x}{\sqrt{x^4-x^2+6}}\,dx = \int \frac{\frac{5}{2}\,du}{u^{1/2}} = \frac{5}{2}\int u^{-1/2}\,du$

$= \frac{5}{2}\left(2u^{1/2}\right) + C$

$= 5\sqrt{x^4-x^2+6} + C$

Let $u = x^4-x^2+6$

Then $du = (4x^3-2x^2)\,dx$

$\frac{1}{2}\,du = (2x^3-x^2)\,dx$

$\frac{5}{2}\,du = (10x^3-5x^2)\,dx$

20. $\displaystyle\int \frac{6u-3}{4u^2-4u+1}\,du = 3\int\frac{(2u-1)\,du}{4u^2-4u+1} = 3\int \frac{\frac{1}{4}\,dw}{w}$

can't call it u...

$= \frac{3}{4}\int \frac{1}{w}\,dw$

Let $w = 4u^2-4u+1$

$= \frac{3}{4}\ln|w| + C$

then $dw = (8u-4)\,du$

$= \frac{3}{4}\ln|4u^2-4u+1| + C$

$\frac{1}{4}\,dw = (2u-1)\,du$

22. $\displaystyle\int \frac{1}{x\ln x}\,dx = \int \frac{1}{\ln x}\cdot\frac{1}{x}\,dx = \int \frac{1}{u}\,du$

Let $u = \ln x$

$du = \frac{1}{x}\,dx$

$= \ln|u| + C$

$= \ln|\ln x| + C$

26. $\displaystyle\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx = 2\int e^u\,du = 2e^u + C = 2e^{\sqrt{x}} + C$

Let $u = x^{1/2}$

$du = \frac{1}{2}x^{-1/2}\,dx$

$2\,du = \frac{1}{\sqrt{x}}\,dx$

Section 5.2 - cont

28. $f'(x) = \frac{2x}{1-3x^2}$. Find $f(x)$ if $f(0)=5$.

$f(x) = \int \frac{2x}{1-3x^2}\,dx = -\frac{1}{3}\int \frac{1}{u}\,du = -\frac{1}{3}\ln|1-3x^2| + C$

$u = 1-3x^2$
$du = -6x\,dx$
$-\frac{1}{3}du = 2x\,dx$

$f(0) = 5 = -\frac{1}{3}\ln 1 + C = C$
$f(x) = -\frac{1}{3}\ln|1-3x^2| + 5$

34. $V'(t) = -960e^{-t/5}$. When $t=0$, $V=5000$. Find $V(10)$.

$V(t) = -960\int e^{-\frac{1}{5}t}\,dt = \frac{-960}{-1/5}e^{-\frac{1}{5}t} + C = 4800e^{-t/5} + C$

$V(0) = 5000 = 4800e^0 + C = 4800 + C$
$200 = C$
$V(t) = 4800e^{-t/5} + 200$
$V(10) = 4800e^{-2} + 200 \approx 849.61$

41. $P(0) = 300$ (working in pennies, no decimals, yay!)
$P'(x) = 3\sqrt{x+1}$. Find $P(8)$.

$P(x) = 3\int (x+1)^{1/2}\,dx = 3\int u^{1/2}\,du = 3(\frac{2}{3}u^{3/2}) + C$
$= 2(x+1)^{3/2} + C$

$u = x+1$
$du = dx$

$P(0) = 2(1)^{3/2} + C = 2 + C = 300$, so $C = 298$
$P(x) = 2(x+1)^{3/2} + 298$
$P(8) = 2(9)^{3/2} + 298 = 2(27) + 298 = 54 + 298$
$= 352.$

So \$3.52 per kilogram