Section 6.1

2. $\int_{-1}^{0} (-3x^5 - 3x^2 + 2x + 5)\,dx = \left[\frac{-1}{2}x^6 - x^3 + x^2 + 5x\right]_{-1}^{0}$
$= 0 - \left(\frac{-1}{2} + 1 + 1 - 5\right) = 7/2$

6. $\int_0^{\ln 2} (e^t - e^{-t})\,dt = \left[e^t + e^{-t}\right]_0^{\ln 2}$
$= \left(e^{\ln 2} + e^{-\ln 2}\right) - \left(e^0 + e^0\right)$
$= 2 + \frac{1}{2} - 2 = 1/2$

10. $\int_{-3}^{0} (2x+6)^4\,dx = \int_{x=-3}^{x=0} u^4 \left(\frac{1}{2}du\right) = \frac{1}{2}\int_{x=-3}^{x=0} u^4\,du$

$u = 2x+6$ $\quad = \frac{1}{2}\left(\frac{1}{5}\right)u^5 \Big|_{x=-3}^{x=0} = \frac{1}{10}(2x+6)^5\Big|_{-3}^{0}$

$du = 2dx$

$\frac{1}{2}du = dx$ $\quad = \frac{1}{10}(6)^5 - \frac{1}{10}(0)^5 = \frac{6^5}{10} = \frac{26136}{10} = \frac{13068}{5}$

notice we left the x-values alone, and filled into x at the end.

14. $\int_0^1 \frac{6x}{x^2+1}\,dx = \int_1^2 \frac{6(1/2\,du)}{u} = 3\ln|u|\Big|_1^2$

$u = x^2+1$ $\quad$ if $x=0, u=1$ $\quad = 3\ln 2 - 3\ln 1$

$du = 2x\,dx$ $\quad$ if $x=1, u=2$ $\quad = 3\ln 2$

$\frac{1}{2}du = x\,dx$

Notice we changed x-values into u-values, and filled into u at the end. Either way works.

18. $\int_e^{e^2} \frac{1}{x\ln x}\,dx = \int_{x=e}^{x=e^2} \frac{1}{u}\,du = \ln|u|\Big|_{x=e}^{x=e^2} = \ln|\ln x|\Big|_e^{e^2}$

$u = \ln x$ $\quad = \ln|\ln e^2| - \ln|\ln e|$

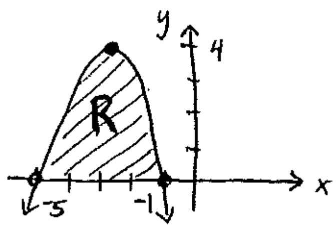
$du = \frac{1}{x}dx$ $\quad = \ln 2 - \ln 1 = \ln 2$

If we switch to u,
$x = e \to u = \ln e = 1$
$x = e^2 \to u = \ln e^2 = 2$

$= \int_1^2 \frac{1}{u}\,du = \ln|u|\Big|_1^2 = \ln 2 - \ln 1$
$= \ln 2$

Section 6.1 - (cont)

26. Find the area of the region bounded by $y=-x^2-6x-5$ and the x-axis. $y=-(x^2+6x+5)=-(x+5)(x+1)$



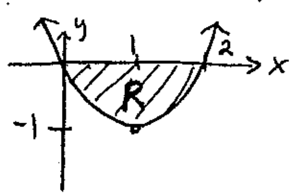
$$\text{Area} = \int_{-5}^{-1} -(x^2+6x+5)\,dx$$

$$= \left[-\left(\tfrac{1}{3}x^3+3x^2+5x\right)\right]_{-5}^{-1}$$

$$= \left(-\left(\tfrac{-1}{3}+3-5\right)\right) - \left(-\left(\tfrac{-125}{3}+75-25\right)\right)$$

$$= \tfrac{7}{3} - \left(\tfrac{-25}{3}\right) = 32/3$$

28. Find the area of the region bounded by $y=x^2-2x$ and the x-axis.



$$\text{Area} = \int_0^2 \left[(\underbrace{\text{top function}}_{y=0}) - (\underbrace{\text{bottom function}}_{y=x^2-2x})\right]dx$$

$$= \int_0^2 (-x^2+2x)\,dx$$

$$= \left(-\tfrac{1}{3}x^3+x^2\right)\Big|_0^2 = \left(-\tfrac{8}{3}+4\right)-(0+0)$$

$$= 4/3$$

32. R is the region between $y=x^3-3x^2$ and $y=x^2+5x$

$y=x^2(x-3)$ $\qquad y=x(x+5)$

Where do they intersect? $x^3-3x^2=x^2+5x$

$$x^3-4x^2-5x=0$$

$$x(x^2-4x-5)=0$$

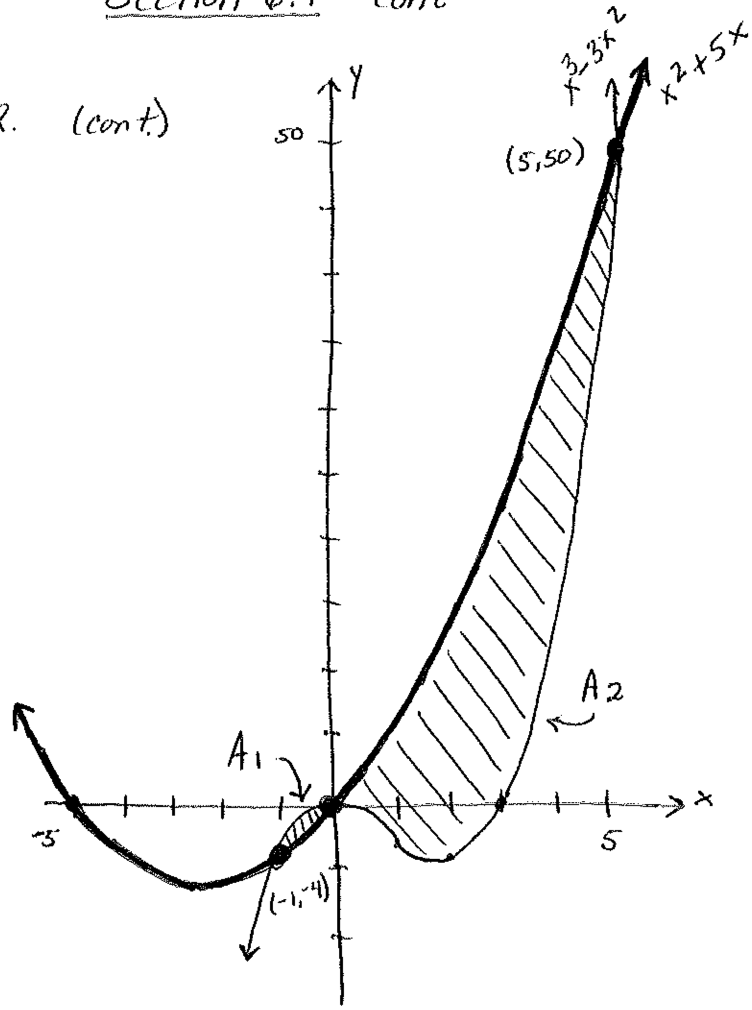
$$x(x-5)(x+1)=0$$

$$(0,0),\ (5,50),\ (-1,-4).$$

Graph and area on next page.

Section 6.1 - cont

32. (cont)



Region has two parts.

$$A = A_1 + A_2 = \int_{-1}^{0} [(x^3-3x^2)-(x^2+5x)]dx + \int_0^5 [(x^2+5x)-(x^3-3x^2)]\,dx$$

$$= \int_{-1}^{0} (x^3-4x^2-5x)\,dx + \int_0^5 (-x^3+4x^2+5x)\,dx$$

$$= \left[\tfrac{1}{4}x^4 - \tfrac{4}{3}x^3 - \tfrac{5}{2}x^2\right]_{-1}^{0} + \left[\tfrac{-1}{4}x^4 + \tfrac{4}{3}x^3 + \tfrac{5}{2}x^2\right]_0^5$$

$$= \left[0 - \left(\tfrac{1}{4}+\tfrac{4}{3}-\tfrac{5}{2}\right)\right] + \left[-\tfrac{625}{4} + \tfrac{500}{3} + \tfrac{125}{2}\right] 0]$$

$$= \frac{-626}{4} + \frac{496}{3} + \frac{130}{2}$$

$$= -\frac{313}{2} + \frac{992}{6} + \frac{390}{6}$$

$$= -\frac{939}{6} + \frac{1382}{6} = \frac{443}{6}$$