NAME KEY

Math 12
Test 1
Fall 2011

You have 50 minutes to complete this test. You must *show all work* to receive full credit. Work any 7 of the following 8 problems. Clearly **CROSS OUT** the problem you do not wish me to grade. Each problem is worth 14 points, and you get 2 points for free, for a total of 100 points. If you have any questions, please come to the front and ask.

1. Using the definition of the derivative, find $f'(x)$ if $f(x) = 2x^3 - 2x^2 + 4$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{[2(x+h)^3 - 2(x+h)^2 + 4] - [2x^3 - 2x^2 + 4]}{h}$$

$$= \lim_{h\to 0} \frac{2(\cancel{x^3} + 3x^2h + 3xh^2 + h^3) - 2(\cancel{x^2} + 2xh + h^2) + \cancel{4} - \cancel{2x^3} + \cancel{2x^2} - \cancel{4}}{h}$$

$$= \lim_{h\to 0} \frac{6x^2h + 6xh^2 + 2h^3 - 4xh - 2h^2}{h}$$

$$= \lim_{h\to 0} (6x^2 + 6xh + 2h^2 - 4x - 2h) = 6x^2 - 4x$$

2. Evaluate the following limits. If any of them do not exist, EXPLAIN why not ("because it's undefined" and "denominator is zero" are not sufficient explanations).

(a) $\lim_{x\to 1} \frac{x^2 - 3x + 2}{x^2 + 1} = \frac{1 - 3 + 2}{1 + 1} = \frac{0}{2} = 0$

(b) $\lim_{x\to -2^+} \frac{2x}{4 - x^2}$ fill in $x=-2$, get $\frac{-4}{0}$, so use chart:

$= -\infty$

x	y
-1	$-2/3$
-1.5	$-3/1.75 = -12/7$
-1.9	$-3.8/0.39 \approx -9.74$
-1.99	$\frac{-3.98}{0.0399} \approx -99.75$

(c) $\lim_{x\to 0} \frac{(x+2)^2 - 4}{x} = \lim_{x\to 0} \frac{x^2 + 4x + 4 - 4}{x}$

fill in, get $\frac{0}{0}$ ...

$= \lim_{x\to 0} \frac{x^2 + 4x}{x} = \lim_{x\to 0} (x+4) = 4$

3. The total cost for a manufacturer to produce q units of a product is $C(q)=\frac{1}{6}q^3+642q+400$ dollars. The current level of production is 4 units. Estimate the amount by which the manufacturer should decrease production in order to reduce the total cost by \$130.

If q changes by Δq, then C changes by $\underset{\text{slope}}{C'(q)}\,\underset{\text{distance in "x-direction".}}{\Delta q}$.

$\Delta C \approx C'(4)\,\Delta q$

cost should go down \$130, so we want to find Δq so that $-130 = C'(4)\,\Delta q$.

$C'(q) = \frac{1}{2}q^2+642$

$C'(4) = \frac{1}{2}(16)+642 = 650$. (If q goes from 4 to 5, C goes up 650.)

$-130 = 650\,\Delta q$

$\frac{-130}{650} = \Delta q$. $\Delta q = \frac{-1}{5}$. Production should be decreased by 1/5.

4. Find y' for the following functions (do not simplify):

a) $y=\left(\sqrt{x}-3x+1\right)\left(\sqrt[4]{x}-2\sqrt{x}\right) = \left(x^{1/2}-3x+1\right)\left(x^{1/4}-2x^{1/2}\right)$

$y' = \left(\frac{1}{2}x^{-1/2}-3\right)\left(x^{1/4}-2x^{1/2}\right)+\left(x^{1/2}-3x+1\right)\left(\frac{1}{4}x^{-3/4}-x^{-1/2}\right)$

b) $y=\dfrac{5x^{-4}+x^3+7}{3x^2+x-2}$

$$y' = \frac{\left(-20x^{-5}+3x^2\right)\left(3x^2+x-2\right)-\left(5x^{-4}+x^3+7\right)\left(6x+1\right)}{\left(3x^2+x-2\right)^2}$$

5. A manufacturer sells all q units of a product that are produced. Suppose the price of the product is \$16 per unit, fixed costs for production total \$10,000, and variable cost is given by $8q$. How many units must be produced in order for the manufacturer to break even?

To break even, Revenue = Total cost

(price)(quantity) = variable cost + fixed cost

$$(16)(q) = 8q + 10000$$

$$8q = 10000$$

$$q = \frac{10000}{8} = 1250$$

produce 1250 units in order to breakeven.

6. Find the equation of the line tangent to $f(x) = \dfrac{7x^3 + x}{2\sqrt{x}}$ at the point where $x = 1$.

$$f'(x) = \frac{(21x^2+1)(2\sqrt{x}) - (7x^3+x)(x^{-1/2})}{(2\sqrt{x})^2}$$

$$m = f'(1) = \frac{(22)(2) - (7+1)(1)}{(2)^2} = \frac{44-8}{4} = 9$$

Point : $x=1$, $y = \frac{7+1}{2} = 4$ $(1,4)$

line : $y - 4 = 9(x-1)$ or $y = 9x - 9 + 4$

$y = 9x - 5$

7. Consider the graph of the function $f(x)$ given below.

a) Find $\lim_{x\to 1} f(x)$. $= 2$

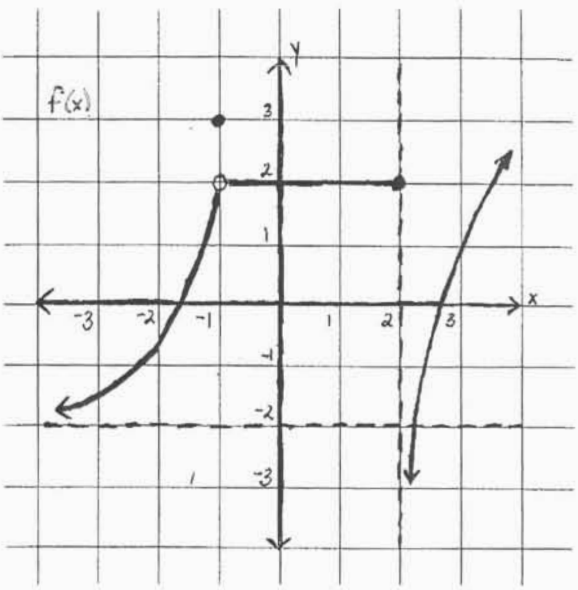
b) Find $\lim_{x\to 2^+} f(x)$. $= -\infty$

c) Find $\lim_{x\to 2^-} f(x)$. $= 2$

d) Find $\lim_{x\to 2} f(x)$. DNE

e) Find $\lim_{x\to -1} f(x)$. $= 2$

f) Find $\lim_{x\to -\infty} f(x)$. $= -2$



8. Fully discuss the continuity of the function $f(x)=\begin{cases}\dfrac{3x}{x-1} & if \quad x\le 2\\ x+2 & if \quad x>2\end{cases}$.

for $x \le 2$, $f(x) = \dfrac{3x}{x-1}$. This is discontinuous at $x=1$.

for $x > 2$, $f(x) = x+2$. This is continuous for all relevant x.

we still need to check for continuity at $x=2$.

There is a point at $\dfrac{3(2)}{2-1} = \dfrac{6}{1} = 6$, $(2,6)$

And there is a hole at $2+2=4$, $(2,4)$.

The point & hole do NOT join up.

Overall f is continuous for all $x \neq 1, 2$.

OR f is continuous on $(-\infty,1)\cup(1,2)\cup(2,\infty)$

OR f is discontinuous at $x=1$, $x=2$, but continuous everywhere else.