NAME KEY

Math 12
Test 1
Spring 2011

You have 50 minutes to complete this test. You must *show all work* to receive full credit. Work any 8 of the following 9 problems. Clearly **CROSS OUT** the problem you do not wish me to grade. Each problem is worth 12 points, and you get 4 points for free, for a total of 100 points. The answers will be posted on the electronic reserves later today.

1. Use the ***definition of the derivative*** to find $f'(x)$ if $f(x)=\frac{1}{x^2}$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{x^2-(x+h)^2}{(x+h)^2x^2}}{h} = \lim_{h\to 0} \frac{x^2-x^2-2xh-h^2}{(x+h)^2x^2}\cdot\frac{1}{h}$$

$$= \lim_{h\to 0} \frac{(-2x-h)h}{(x+h)^2x^2}\cdot\frac{1}{h} = \lim_{h\to 0} \frac{-2x-h}{(x+h)^2x^2}$$

$$= \frac{-2x}{x^4} = -2x^{-3}$$

2. Calculate the following limits.

(a) $\lim_{x\to 1}\left(\frac{1}{x^2}-\frac{1}{x}\right) = \frac{1}{1}-\frac{1}{1} = 0$

(b) $\lim_{x\to 1}\frac{x^2+x-2}{x^2-1}$ fill in $x=1$, get $\frac{0}{0}$, so... $\lim_{x\to 1}\frac{(x+2)(x-1)}{(x+1)(x-1)}$

$$= \lim_{x\to 1}\frac{x+2}{x+1} = 3/2$$

(c) $\lim_{x\to 1}\frac{\sqrt{x}-1}{x-1}$ fill in $x=1$, get $\frac{0}{0}$, so...

$$= \lim_{x\to 1}\frac{(\sqrt{x}-1)(\sqrt{x}+1)}{(x-1)(\sqrt{x}+1)} = \lim_{x\to 1}\frac{x-1}{(x-1)(\sqrt{x}+1)}$$

$$= \lim_{x\to 1}\frac{1}{\sqrt{x}+1} = \frac{1}{2}$$

3. A manufacturer can produce microwaves at a cost of \$80 apiece. If they are sold for x dollars each, $50 - x$ microwaves will be sold each month.

a) Express the monthly profit as a function of the price x.
b) Sketch a graph of this profit function.
c) Estimate the price that will result in the highest profit.

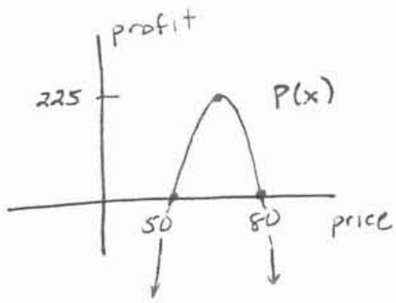
a) Profit = Revenue - cost = (price)(quantity) - cost

$P = (x)(50-x) - (80)(50-x) = 50x - x^2 - 4000 + 80x$

$P = -x^2 + 130x - 4000 = -(x^2 - 130x + 4000)$

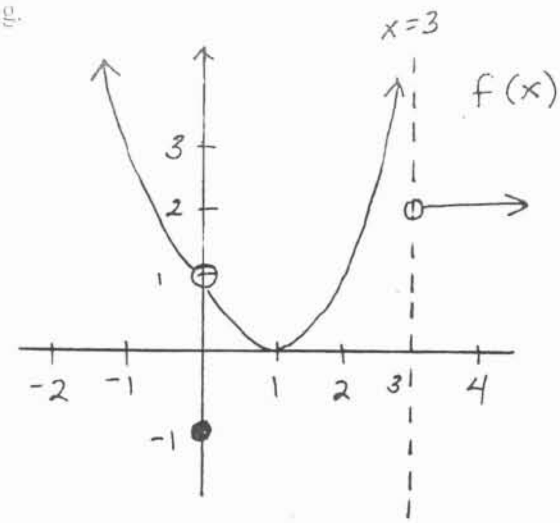
$P = -(x-80)(x-50)$

b)



c) vertex is when x = price = \$65 each.
(this will give total profit of \$225)

4. Use the given graph to determine the following.



a) $\lim\limits_{x \to 4} f(x) = 1$

b) $\lim\limits_{x \to 3^+} f(x) = 1$

c) $\lim\limits_{x \to 3^-} f(x) = \infty$

d) $\lim\limits_{x \to 3} f(x)$ DNE

e) $\lim\limits_{x \to 0} f(x) = 1$

f) At what x-values is $f(x)$ discontinuous?

Dicontinuous at $x=0$ and at $x=3$

5. Find $f'(x)$ for the following functions. DO NOT simplify!

(a) $f(x)=\frac{2}{3x^2}-\frac{x}{3}+\frac{4}{5}+\frac{x+1}{x} = \frac{2}{3}x^{-2}-\frac{1}{3}x+\frac{4}{5}+1+x^{-1}$

$f'(x) = -\frac{4}{3}x^{-3}-\frac{1}{3}-x^{-2}$

(b) $f(x)=(x^2+2)(x+\sqrt{x}) = (x^2+2)(x+x^{1/2})$

$f'(x) = (2x)(x+x^{1/2})+(x^2+2)(1+\frac{1}{2}x^{-1/2})$

(c) $f(x)=\frac{x+7x^{-4}+3}{5-2x^2+3x}$

$$f'(x) = \frac{(1-28x^{-5})(5-2x^2+3x)-(x+7x^{-4}+3)(-4x+3)}{(5-2x^2+3x)^2}$$

6. Find the equation of the line tangent to the graph of $f(x)=\frac{x+\sqrt{x}}{x\sqrt{x}}$ at the point where $x=1$.

point: $x=1$, $y=\frac{1+\sqrt{1}}{1\sqrt{1}}=\frac{2}{1}=2$ $\quad(1,2)$

slope: $f'(x) = \frac{(1+\frac{1}{2}x^{-1/2})(x\sqrt{x})-(x+\sqrt{x})(\frac{3}{2}x^{1/2})}{x^3}$

$m=f'(1) = \frac{(1+1/2)(1)-(2)(3/2)}{1} = \frac{3}{2}-3=-3/2$

Line: $y-2=-\frac{3}{2}(x-1)$

7. Find the equation of the line perpendicular to the line $x+3y=5$ which contains the point $(-2,3)$.

$x+3y=5$
$3y=-x+5$
$y=\frac{-1}{3}x+\frac{5}{3}$
old slope $=-1/3$,
perpendicular,
so our $m=3$.

$y-3=3(x+2)$

8. Suppose x units of a product are produced and all units will be sold if the price is $p(x)=25-\frac{1}{3}x$ dollars per unit.

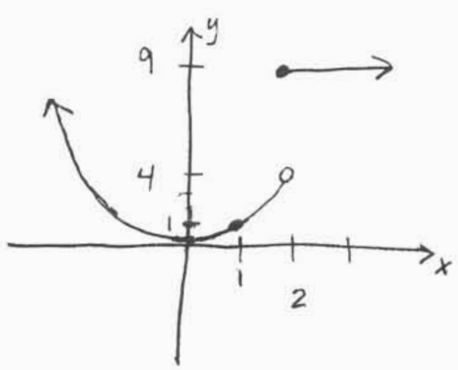
(a) Find the revenue function.
(b) Use the marginal revenue function to ***estimate*** the revenue derived from the sale of the 9^{th} unit.
(c) Find the ***actual*** revenue derived from the sale of the 9^{th} unit.

a) Rev = price · quantity $=(25-\frac{1}{3}x)x$
$R=25x-\frac{1}{3}x^2$

b) $R'=25-\frac{2}{3}x$
$R'(8)=25-\frac{16}{3}=\frac{59}{3}$ dollars extra from sale of 9th unit. (approx)

c) actual revenue from 9th unit $=R(9)-R(8)$
$=[25(9)-\frac{1}{3}(81)]-[25(8)-\frac{1}{3}(64)]$
$=25-\frac{81}{3}+\frac{64}{3}=\frac{75}{3}-\frac{17}{3}=\frac{52}{3}$ dollars

9. Sketch the graph of $f(x)=\begin{cases} x^2 & x<2 \\ 9 & x\geq 2\end{cases}$ and describe the continuity of this function.



f(x) is continuous on $(-\infty,2)\cup(2,\infty)$.
f(x) is discontinuous at $x=2$ because

$\lim_{x\to 2^-} f(x)=4$
$\lim_{x\to 2^+} f(x)=9$
$\Big\}$ not equal.