NAME KEY

Math 12
Test 1
Fall 2010

You have 50 minutes to complete this test. You must *show all work* to receive full credit. Work any 7 of the following 8 problems. Clearly **CROSS OUT** the problem you do not wish me to grade. Each problem is worth 14 points, and you get 2 points for free, for a total of 100 points. If you have any questions, please come to the front and ask.

1. Using the definition of the derivative, find $f'(x)$ if $f(x) = \dfrac{1}{x-2}$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\frac{1}{x+h-2} - \frac{1}{x-2}}{\frac{h}{1}}$$

$$= \lim_{h\to 0} \left(\frac{(x-2)-(x+h-2)}{(x+h-2)(x-2)}\right)\left(\frac{1}{h}\right)$$

$$= \lim_{h\to 0} \frac{-h}{(x+h-2)(x-2)} \cdot \frac{1}{h}$$

$$= \lim_{h\to 0} \frac{-1}{(x+h-2)(x-2)}$$

$$= \frac{-1}{(x-2)^2}$$

2. Evaluate the following limits. If any of them do not exist, EXPLAIN why not ("because it's undefined" and "denominator is zero" are not sufficient explanations).

(a) $\displaystyle\lim_{x\to 4} \frac{9-x}{3-\sqrt{x}} = \frac{9-4}{3-\sqrt{4}} = \frac{5}{1} = 5$

(b) $\displaystyle\lim_{x\to 2^+} \frac{x+1}{x-2}$

fill in, get $\frac{3}{0}$, must use chart:

| x | y |
|---|---|
| 3 | $4/1 = 4$ |
| 2.5 | $3.5/.5 = 7$ |
| 2.1 | $3.1/.1 = 31$ |
| 2.01 | $3.01/.01 = 301$ |

as $x \to 2^+$, $y \to \infty$.

$$\lim_{x\to 2^+} \frac{x+1}{x-2} = \infty$$

(c) $\displaystyle\lim_{x\to -2} \frac{x^2-x-6}{x^2+3x+2} = \lim_{x\to -2} \frac{(x+2)(x-3)}{(x+2)(x+1)} = \lim_{x\to -2} \frac{x-3}{x+1}$

$$= \frac{-5}{-1} = 5$$

3. The supply of a product is given by $S(p) = p - 10$ and the demand is given by $D(p) = \dfrac{5600}{p}$ when the price is $p$.

a) Find the equilibrium price and the corresponding number of units supplied and demanded. $\hookrightarrow S(p) = D(p)$

$p - 10 = \dfrac{5600}{p}$

$p^2 - 10p = 5600$

$p^2 - 10p - 5600 = 0$

$(p-80)(p+70) = 0$

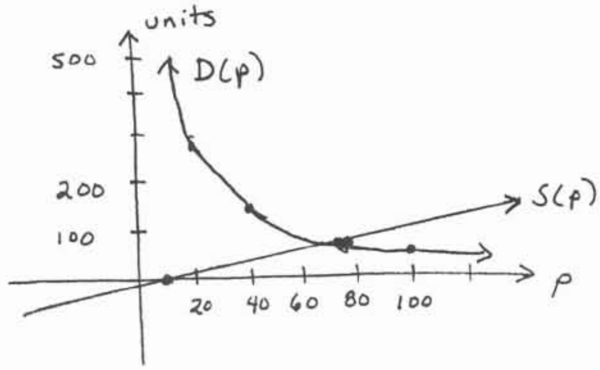
$p = 80$

$p = -70$ No!

$S(80) = 80 - 10 = 70$ units

$D(80) = \dfrac{5600}{80} = 70$ units

b) Draw the supply and demand curves on the given set of axes.




c) Where does the supply curve cross the $p$-axis? Describe the economic significance of this point.

$S(p) = 0 = p - 10$ at $p = 10$

The point (10,0) on the supply curve means that if the price is \$10, no units will be supplied. \$10 is the minimum price to support production of this product.

4. Find $y'$ for the following functions (do not simplify) :

a) $y = (x^3 - 2x + 3)(x^{-2} + 4x^{-3})$

$y' = (3x^2 - 2)(x^{-2} + 4x^{-3}) + (x^3 - 2x + 3)(-2x^{-3} - 12x^{-4})$

b) $y = x\sqrt{x} + \dfrac{4}{3x^2} = x^{3/2} + \dfrac{4}{3}x^{-2}$

$y' = \dfrac{3}{2}x^{1/2} - \dfrac{8}{3}x^{-3}$

5. Suppose the total cost to produce $x$ units of a product is $C(x) = \frac{1}{3}x^2 + 65$.

a) Use marginal analysis to ***estimate*** the cost to produce the 7th unit.

$$C'(x) = \frac{2}{3}x$$

cost to produce 7th unit $\approx C'(6)$

$$\approx \frac{2}{3}(6)$$

$$\approx \$4$$

b) What is the ***actual*** cost to produce the 7th unit?

Actual cost to produce 7th unit $= C(7) - C(6)$

$$= \left(\frac{49}{3} + 65\right) - \left(\frac{36}{3} + 65\right)$$

$= \frac{13}{3}$, about \$4.33

6. Find the equation of the line tangent to $f(x) = \frac{\sqrt{x}}{x+1}$ at the point where $x = 4$.

<u>point</u>: $x = 4$ $\quad (4, 2/5)$

$$y = \frac{\sqrt{4}}{4+1} = \frac{2}{5}$$

<u>Slope</u>: $f'(x) = \dfrac{\left(\frac{1}{2}x^{-1/2}\right)(x+1) - (\sqrt{x})(1)}{(x+1)^2}$

$$m = f'(4) = \frac{\left(\frac{1}{2}\right)\left(\frac{1}{\sqrt{4}}\right)(4+1) - (\sqrt{4})}{(4+1)^2} = \frac{\frac{5}{4} - 2}{25} = \frac{\frac{-3}{4}}{25}$$

$$m = \frac{-3}{100}$$

<u>line</u>: $y - \frac{2}{5} = \frac{-3}{100}(x - 4)$

$$y = \frac{-3}{100}x + \frac{3}{25} + \frac{2}{5}$$

$$y = \frac{-3}{100}x + \frac{13}{25}$$

7. Consider the graph of the function $f(x)$ given below.

(a) Find $\lim\limits_{x \to -2^-} f(x)$. $= 3$

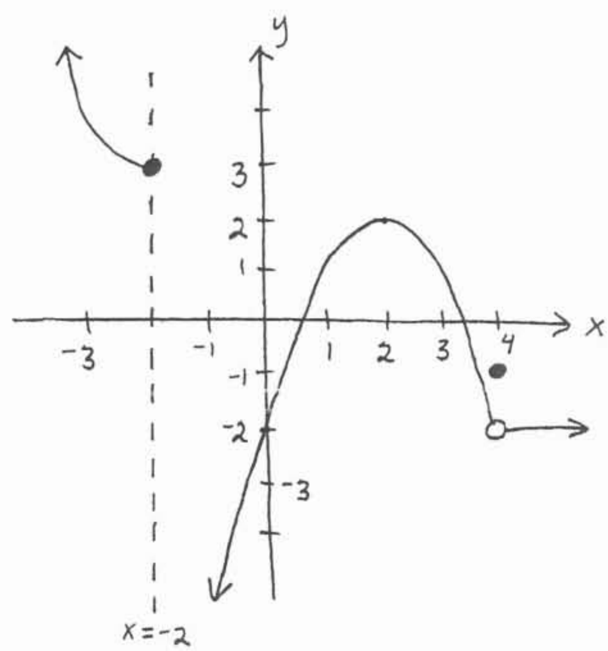
(b) Find $\lim\limits_{x \to -2^+} f(x)$. $= -\infty$

(c) Find $\lim\limits_{x \to -2} f(x)$. $= ?$ DNE

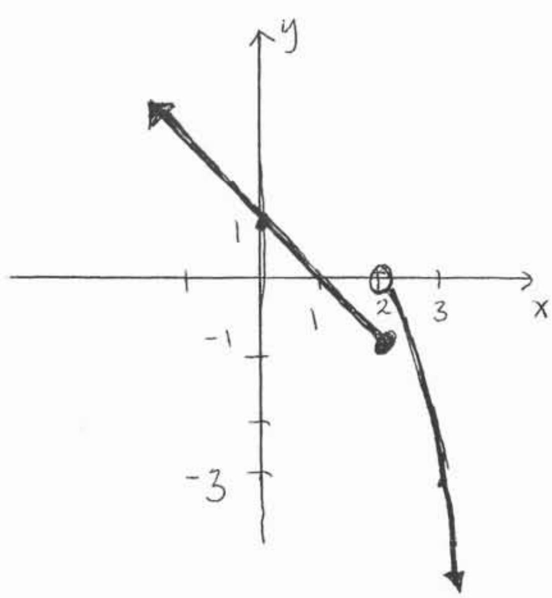
(d) Find $\lim\limits_{x \to 0} f(x)$. $= -2$

(e) Find $\lim\limits_{x \to 4} f(x)$. $= -2$

(f) Find $\lim\limits_{x \to -\infty} f(x)$. $= \infty$




8. Sketch the graph of the function $f(x) = \begin{cases} 1-x & \text{if } x \le 2 \\ 2x - x^2 & \text{if } x > 2 \end{cases}$. Fully discuss the continuity of this function.




f is continuous at all values of x except at $x = 2$. (polynomials)

$\lim\limits_{x \to 2^-} f(x) = \lim\limits_{x \to 2^-} (1-x)$

$= 1 - 2 = -1$

$\lim\limits_{x \to 2^+} f(x) = \lim\limits_{x \to 2^+} (2x - x^2)$

$= 4 - 4 = 0$

$\lim\limits_{x \to 2} f(x)$ DNE, so $f(x)$ is not continuous at $x = 2$.