NAME KEY

Math 12
Test 1
Fall 2012

You have 50 minutes to complete this test. You must *show all work* to receive full credit. Work any 7 of the following 8 problems. Clearly **CROSS OUT** the problem you do not wish me to grade. Each problem is worth 14 points, and you get 2 points for free, for a total of 100 points. If you have any questions, please come to the front and ask.

1. Using the definition of the derivative, find $f'(x)$ if $f(x) = \sqrt{3x} - 4$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\sqrt{3(x+h)} - \cancel{4} - (\sqrt{3x} - \cancel{4})}{h}$$

$$= \lim_{h\to 0} \frac{\sqrt{3(x+h)} - \sqrt{3x}}{h} \cdot \frac{\sqrt{3(x+h)} + \sqrt{3x}}{\sqrt{3(x+h)} + \sqrt{3x}}$$

$$= \lim_{h\to 0} \frac{3(x+h) - 3x}{h\left(\sqrt{3(x+h)} + \sqrt{3x}\right)} = \lim_{h\to 0} \frac{3\cancel{h}}{\cancel{h}\left(\sqrt{3(x+h)} + \sqrt{3x}\right)}$$

$$= \frac{3}{\sqrt{3x} + \sqrt{3x}} = \frac{3}{2\sqrt{3x}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{2\sqrt{x}}$$

2. Evaluate the following limits. If any of them do not exist, EXPLAIN why not ("because it's undefined" and "denominator is zero" are not sufficient explanations).

(a) $\lim_{x\to 2^+} \frac{x+3}{x^2+x-6} = \lim_{x\to 2^+} \frac{x+3}{(x+3)(x-2)} = \lim_{x\to 2^+} \frac{1}{x-2} = \infty$

plug in, get $\frac{5}{0}$...

x	y
3	1
2.5	$\frac{1}{1/2} = 2$
2.1	$1/_{1/10} = 10$
2.01	$\frac{1}{1/100} = 100$

(b) $\lim_{x\to 4} \frac{\sqrt{x}-2}{4-x} = \lim_{x\to 4} \frac{(\sqrt{x}-2)(\sqrt{x}+2)}{(4-x)(\sqrt{x}+2)} = \lim_{x\to 4} \frac{x-4}{(4-x)(\sqrt{x}+2)}$

plug in, get $\frac{0}{0}$ ...

$$= \lim_{x\to 4} \frac{-1}{\sqrt{x}+2} = \frac{-1}{2+2} = \frac{-1}{4}$$

(c) $\lim_{x\to 3} \frac{x^2-2x-3}{x^2+2x-3} = \frac{9-6-3}{9+6-3} = \frac{0}{12} = 0$

3. Suppose that x units of a product will be sold if the price is set at $p(x)=\frac{50000-x}{25000}$. Suppose the total cost for a manufacturer to produce x units of the product is $C(x)=2100+0.25x$ dollars.

a) Find an equation for Revenue.

Revenue = price • quantity

$$R(x)=\frac{50000-x}{25000}\cdot x=\frac{50000}{25000}x-\frac{x}{25000}x=2x-\frac{1}{25000}x^2$$

b) Find an equation for Profit.

Profit = Revenue − Cost

$$P(x)=\left(2x-\frac{1}{25000}x^2\right)-(2100+0.25x)=1.75x-\frac{1}{25000}x^2-2100$$

c) Suppose 15000 units are currently produced, and the company's goal is to have the highest possible profit. Use marginal analysis to determine whether or not production should be increased.

$$P'(x)=1.75-\frac{1}{12500}x$$

$$P'(15000)=1.75-\frac{15000}{12500}=1.75-1.2=0.55$$

This means that production and sale of the next unit will increase profit, since P' is positive. The company should increase production.

4. Find the equation of the line parallel to $4x-3y=2$ that goes through the point $(5,-2)$.

$$4x-3y=2$$
$$-3y=-4x+2$$
$$y=\frac{4}{3}x-\frac{2}{3}$$

$m=\frac{4}{3}$, parallel, so use same slope.

Line: $y+2=\frac{4}{3}(x-5)$ ← ok to stop here.

$$y=\frac{4}{3}x-\frac{20}{3}-\frac{6}{3}$$
$$y=\frac{4}{3}x-\frac{26}{3}$$

5. Find y' for the following functions (do not simplify) :

a) $y=\left(x^{-2}-x^{-3}\right)\left(3x^{-1}+4x^{-4}\right)$

$$y' = (-2x^{-3}+3x^{-4})(3x^{-1}+4x^{-4}) + (x^{-2}-x^{-3})(-3x^{-2}-16x^{-5})$$

b) $y=5x^4-\dfrac{3}{4x^2}+6\sqrt[3]{x^2}-\dfrac{1}{x}+\dfrac{2x^3+5}{x^2}$

ok to leave as a fraction & do quotient rule.

$$y = 5x^4 - \frac{3}{4}x^{-2} + 6x^{2/3} - x^{-1} + \overbrace{(2x^3+5)(x^{-2})}$$

$$y = 5x^4 - \frac{3}{4}x^{-2} + 6x^{2/3} - x^{-1} + 2x + 5x^{-2}$$

$$y' = 20x^3 + \frac{3}{2}x^{-3} + 4x^{-1/3} + x^{-2} + \underbrace{2 - 10x^{-3}}$$

using quotient rule instead: $\dfrac{(6x^2)(x^2)-(2x^3+5)(2x)}{x^4}$

6. Find the equation of the line tangent to the graph of $f(x)=\dfrac{\sqrt{x}+1}{2x-3}$ at the point where $x=1$.

$$f(x) = \frac{x^{1/2}+1}{2x-3}$$

Point : $x=1$ $\quad (1,-2)$

$$y = f(1) = \frac{1+1}{2-3} = -2$$

Slope : $$f'(x) = \frac{(\frac{1}{2}x^{-1/2})(2x-3) - (x^{1/2}+1)(2)}{(2x-3)^2}$$

$$m = f'(1) = \frac{(\frac{1}{2})(-1)-(2)(2)}{(-1)^2} = \frac{-1}{2} - 4 = \frac{-9}{2}$$

line : $y+2 = \frac{-9}{2}(x-1)$ ⟵ ok to stop here

$$y = \frac{-9}{2}x + \frac{9}{2} - \frac{4}{2}$$

$$y = \frac{-9}{2}x + \frac{5}{2}$$

7. Consider the graph of the function $f(x)$ given below.

a) Find $\lim_{x\to 0} f(x)$. $=1$

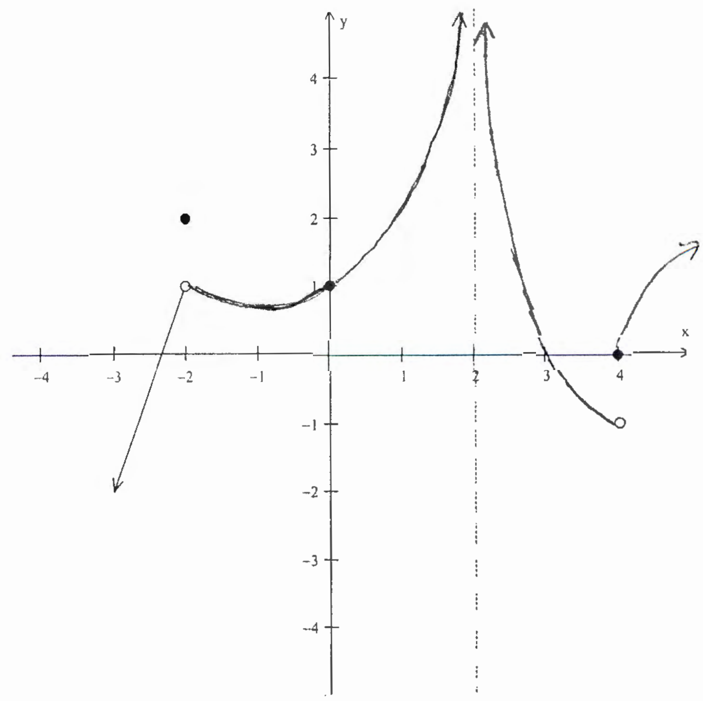
b) Find $\lim_{x\to -2} f(x)$. $=1$

c) Find $\lim_{x\to 2} f(x)$. $=\infty$

d) Find $\lim_{x\to 4^-} f(x)$. $=-1$

e) Find $\lim_{x\to 4^+} f(x)$. $=0$

f) Find $\lim_{x\to 4} f(x)$. DNE



8. For what value of A will the function $f(x)=\begin{cases} x^2-2x+1 & \text{if } x\le 3 \\ 2Ax+3 & \text{if } x>3 \end{cases}$ be continuous at $x=3$? Show all your reasoning.

Point at $x=3$, $y=3^2-2(3)+1=9-6+1=4$

$(3,4)$

Hole at $x=3$, $y=2A(3)+3$

$y=6A+3$

$(3, 6A+3)$.

we need the point to join up with the hole, so $4=6A+3$

$1=6A$

$A=1/6$