NAME KEY

Math 12
Test 1
Spring 2012

You have 50 minutes to complete this test. You must *show all work* to receive full credit. Work any 7 of the following 8 problems. Clearly **CROSS OUT** the problem you do not wish me to grade. Each problem is worth 14 points, and you get 2 points for free, for a total of 100 points. If you have any questions, please come to the front and ask.

1. Using the definition of the derivative, find $f'(x)$ if $f(x)=\frac{1}{x^2}$.

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{\frac{1}{(x+h)^2}-\frac{1}{x^2}}{h}$$

$$=\lim_{h\to 0}\frac{x^2-(x+h)^2}{(x+h)^2\cdot x^2}\cdot\frac{1}{h}=\lim_{h\to 0}\frac{x^2-x^2-2xh-h^2}{(x+h)^2x^2h}$$

$$=\lim_{h\to 0}\frac{h(-2x-h)}{h(x+h)^2x^2}=\lim_{h\to 0}\frac{-2x-h}{(x+h)^2x^2}=\frac{-2x}{x^4}=\frac{-2}{x^3}$$

2. Evaluate the following limits. If any of them do not exist, EXPLAIN why not ("because it's undefined" and "denominator is zero" are not sufficient explanations).

(a) $\lim_{x\to 3}\frac{9-x^2}{x-3}=\lim_{x\to 3}\frac{(3-x)(3+x)}{x-3}=\lim_{x\to 3}\frac{-(x-3)(3+x)}{x-3}=-6$

(fill in, get $\frac{0}{0}$)

(b) $\lim_{x\to 0^+}\left(x-\frac{1}{x}\right)=\lim_{x\to 0+}\frac{x^2-1}{x}$ fill in, get $\frac{-1}{0}$ ...

$=-\infty$, see chart

x	y
1	$\frac{0}{1}=0$
0.5	$-3/2$
0.1	-9.9
0.01	-99.99

(c) $\lim_{x\to 1}\frac{2x+3}{x+1}=\frac{2(1)+3}{1+1}=\frac{5}{2}$

3. Suppose that George is the business manager for a company that manufactures digital cameras. If x hundred cameras are produced, they can all be sold if the price is set at $p(x) = 300 - 0.0035x^2$ dollars. The cost to produce x hundred cameras is $C(x) = 200 - 0.07x^2 + 275x$.

a) Find a function for Profit.

Profit = Revenue − Cost = (price)(quantity) − Cost

$P(x) = (300 - 0.0035x^2)(x) - (200 - 0.07x^2 + 275x)$

$P(x) = 300x - 0.0035x^3 - 200 + 0.07x^2 - 275x$

b) Find a function for Marginal Profit. $P(x) = -0.0035x^3 + 0.07x^2 + 25x - 20$

Marginal Profit $= P'(x) = -0.0105x^2 + 0.14x + 25$

c) Suppose the current level of production is $x = 10$ (1000 cameras). Based on the marginal profit at this level of production, should George recommend increasing or decreasing production in order to increase profit?

$P'(10) = -0.0105(100) + 0.14(10) + 25$

$= -1.05 + 1.4 + 25$

$= \$25.35$ expected increase in P if x goes from 10 to 11.

Since $P'(10) > 0$, making the next hundred cameras will increase profit, so YES, recommend increasing production.

4. Find $f'(x)$ (do not simplify!) if :

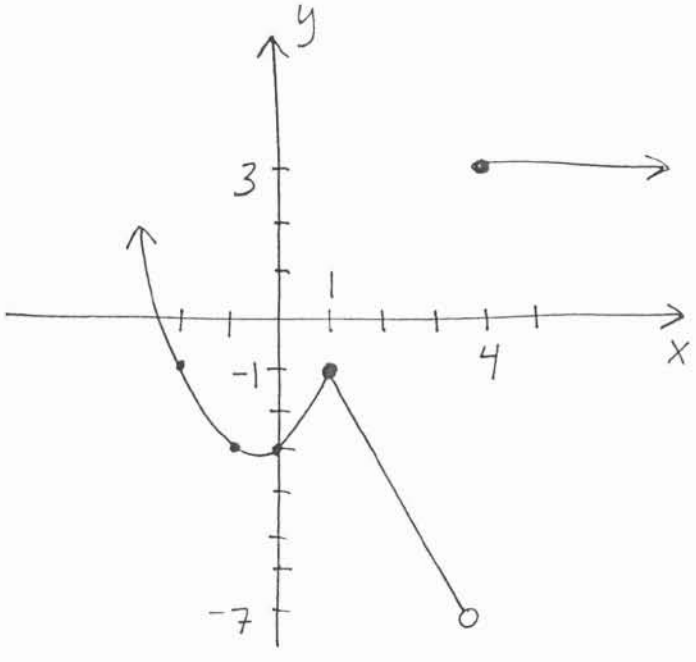
a) $f(x) = \dfrac{x}{x^2 - 1} + \dfrac{4 - x}{x^2 + 1}$

$$f'(x) = \frac{(1)(x^2-1) - (x)(2x)}{(x^2-1)^2} + \frac{(-1)(x^2+1) - (4-x)(2x)}{(x^2+1)^2}$$

b) $f(x) = \left(2\sqrt[3]{x} + 7x^4 - 6\right)\left(x^{-3} + 2x - \pi\right) = (2x^{1/3} + 7x^4 - 6)(x^{-3} + 2x - \pi)$

$$f'(x) = \left(\frac{2}{3}x^{-2/3} + 28x^3\right)(x^{-3} + 2x - \pi) + (2x^{1/3} + 7x^4 - 6)(-3x^{-4} + 2)$$

5. Sketch a nice big graph of $f(x)=\begin{cases} x^2+x-3 & x<1 \\ 1-2x & 1\le x<4 \\ 3 & 4\le x \end{cases}$. Be sure to clearly label points and axes. Under your graph, list the interval(s) where $f(x)$ is continuous.



f(x) is continuous on $(-\infty, 4) \cup (4, \infty)$

6. Find the equation of the line tangent to $f(x)=\dfrac{(x^2+x-3)(4-x)}{2x-1}$ at the point where $x=1$.

Point : $x=1$

$y=\dfrac{(1+1-3)(4-1)}{2-1}=\dfrac{-1\cdot 3}{1}=-3$

$(1,-3)$

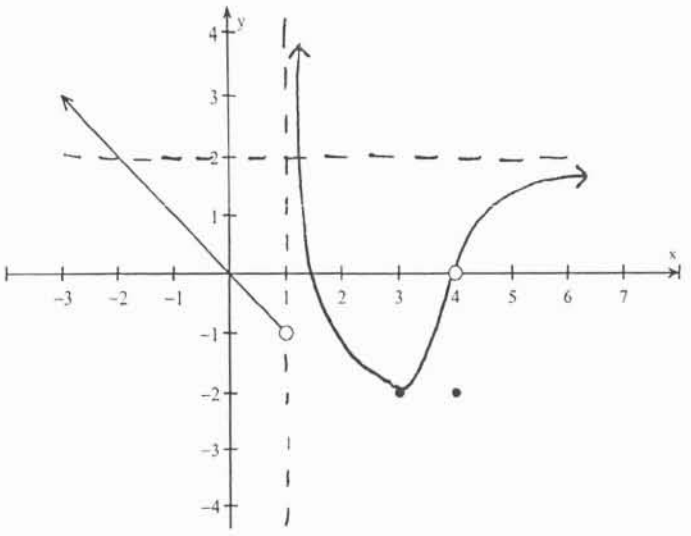
Slope : $f'(x)=\dfrac{[(2x+1)(4-x)+(x^2+x-3)(-1)](2x-1)-(x^2+x-3)(4-x)(2)}{(2x-1)^2}$

$m=f'(1)=\dfrac{[(3)(3)+(-1)(-1)](1)-(-1)(3)(2)}{1}$

$=10+6=16$

Line : $y+3=16(x-1)$

7. Consider the graph of the function $f(x)$ given below.



(a) Find $\lim_{x\to 0} f(x)$. $= 0$

(b) Find $\lim_{x\to 1^-} f(x)$. $= -1$

(c) Find $\lim_{x\to 1^+} f(x)$. $= \infty$

(d) Find $\lim_{x\to -1} f(x)$. DNE

(e) Find $\lim_{x\to 4} f(x)$. $= 0$

(f) Find $\lim_{x\to +\infty} f(x)$. $= 2$

8. Find the equation of the line parallel to $2x + y = 3$ which contains the point $(5,4)$.

Old Line: $2x+y=3$

$y = -2x+3$

$m_{old} = -2$

New Line: $m = -2$ through $(5,4)$

$y - 4 = -2(x-5)$ $\qquad y = -2x+10+4$

$y = -2x+14$