NAME KEY

Math 12
Test 1
Spring 2014

You have 50 minutes to complete this test. You must *show all work* to receive full credit. Work any 7 of the following 8 problems. Clearly **CROSS OUT** the problem you do not wish me to grade. Each problem is worth 14 points, and you get 2 points for free, for a total of 100 points. If you have any questions, please come to the front and ask.

1. Using the definition of the derivative, find $f'(x)$ if $f(x) = 4 + \sqrt{x}$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(4 + \sqrt{x+h}) - (4 + \sqrt{x})}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}} = \lim_{h \to 0} \frac{\cancel{x} + h - \cancel{x}}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}} = \frac{1}{\sqrt{x} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$$

2. Evaluate the following limits. If any of them do not exist, EXPLAIN why not ("because it's undefined" and "denominator is zero" are not sufficient explanations).

(a) $\lim_{x \to 4} \frac{4 - x}{\sqrt{x} - 2} \cdot \frac{\sqrt{x} + 2}{\sqrt{x} + 2} = \lim_{x \to 4} \frac{(4 - x)(\sqrt{x} + 2)}{x - 4} = \lim_{x \to 4} \frac{-\cancel{(x-4)}(\sqrt{x} + 2)}{\cancel{x-4}}$

$$= \lim_{x \to 4} -(\sqrt{x} + 2) = -(2 + 2) = -4$$

(b) $\lim_{x \to 3^+} \frac{x + 3}{x^2 - 9} = \lim_{x \to 3^+} \frac{x + 3}{(x+3)(x-3)} = \lim_{x \to 3^+} \frac{1}{x - 3} = \infty$

x	y
4	1
3.5	$\frac{1}{.5} = 2$
3.1	$\frac{1}{.1} = 10$
3.01	$\frac{1}{.01} = 100$
	$\downarrow$
	∞

(c) $\lim_{x \to 2} \frac{x - 2}{x^2 + 3x - 1} = \frac{0}{4 + 6 - 1} = \frac{0}{9} = 0$

3. Suppose the total cost to produce x snowboards is given by $C(x) = 1000 + 100x - 0.25x^2$ dollars.

a) Using marginal analysis, ***estimate*** how much the total cost willincrease if the production increases from 100 to 102 snowboards.

$c'(x) = 100 - 0.5x$

Slope is $c'(100) = 100 - 0.5(100) = 100 - 50 = 50$

$c'(100) = 50$ = amt cost goes up for the 101^{st} snowboard.

To go from $100 \rightarrow 102$, we have to go up two slopes.

The cost increase is approximately $2(50) = \boxed{\$100}$

b) Find the ***exact*** amount of the cost increase.

exact cost increase is

$$C(102) - C(100) = [1000 + 100(102) - 0.25(102)^2] - [1000 + 100(100) - 0.25(100)^2]$$

$$= 10200 - 2601 - 10000 + 2500$$

$$= \boxed{\$99}$$

4. Find $f'(x)$ (do not simplify!) if :

a) $f(x) = \dfrac{2x^2 - 7}{4x + 3}$

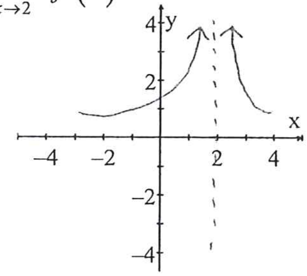
$$f'(x) = \frac{(4x)(4x+3) - (2x^2-7)(4)}{(4x+3)^2}$$

b) $f(x) = (2x^4 - 3x^3 + x - 5)(x^2 - x + 5)$

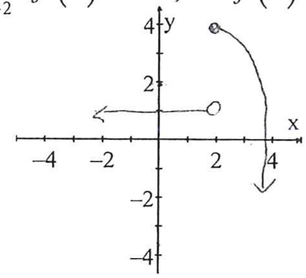
$$f'(x) = (8x^3 - 9x^2 + 1)(x^2 - x + 5) + (2x^4 - 3x^3 + x - 5)(2x - 1)$$

5. For each part below, sketch the graph of a function $f(x)$ which satisfies the given condition(s). Although there may be many graphs that will work, only show ONE as your solution.

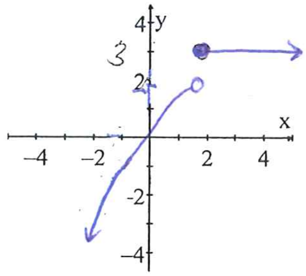
a) $\lim_{x\to 2} f(x) = \infty$



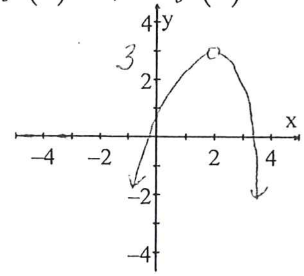
b) $\lim_{x\to 2} f(x)$ DNE, but $f(2) = 4$



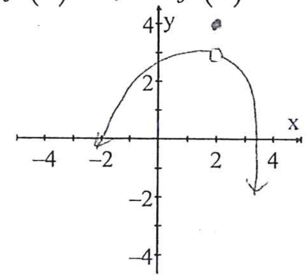
c) $\lim_{x\to 2} f(x)$ DNE, but $\lim_{x\to 2^+} f(x) = 3$



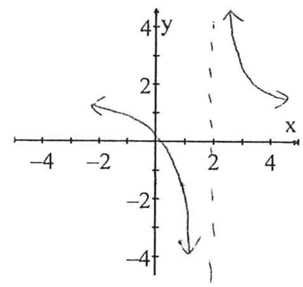
d) $\lim_{x\to 2} f(x) = 3$, but $f(2)$ is undefined



e) $\lim_{x\to 2} f(x) = 3$, but $f(2) = 4$



f) $\lim_{x\to 2^+} f(x) = \infty$ and $\lim_{x\to 2^-} f(x) = -\infty$



6. Find the equation of the line tangent to $y = 5\sqrt{x} + 2x^3 - 4x - \frac{1}{x^2} + 3$ at the point where $x = 1$.

$y = 5x^{1/2} + 2x^3 - 4x - x^{-2} + 3$

Slope: $y' = \frac{5}{2}x^{-1/2} + 6x^2 - 4 + 2x^{-3}$

$m = y'(1) = \frac{5}{2} + 6 - 4 + 2$

$= \frac{5}{2} + 4 = \frac{13}{2}$

Point: $x = 1,\ y = 5 + 2 - 4 - 1 + 3$ (use original)

$= 5$

$(1, 5)$

Line: $y - 5 = \frac{13}{2}(x - 1)$

or $y = \frac{13}{2}x - \frac{3}{2}$

7. A video production company is planning to produce a set of instructional DVDs. The producer estimates that it will cost \$84000 to shoot the video and \$15 per set to copy and distribute the DVDs. The wholesale price of the DVDs is \$50 per set. Suppose x sets of DVDs are produced.

a) Write an equation for the total cost function.

$C(x) = 84000 + 15x$, where x = # sets produced.

b) Write an equation for the total revenue function.

Revenue = price · quantity

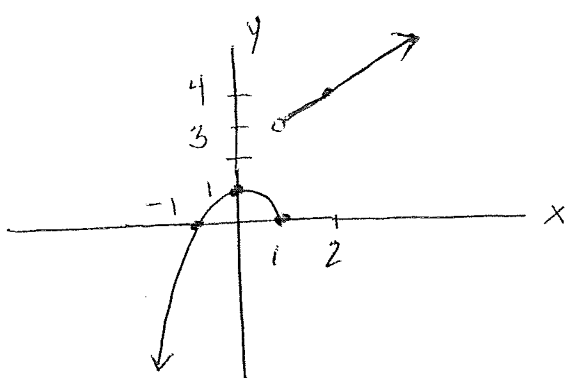
$R(x) = 50x$

c) Find the number of sets that must be produced and sold in order for the company to break even.

To break even, revenue = cost (or profit = 0).

$$R(x) = C(x)$$
$$50x = 84000 + 15x$$
$$35x = 84000$$
$$x = 2400 \text{ sets}$$

8. Sketch a nice big graph of $f(x) = \begin{cases} 1 - x^2 & x \le 1 \\ x + 2 & x > 1 \end{cases}$. Be sure to clearly label points and axes. Under your graph, list the interval(s) where $f(x)$ is continuous.



$f(x)$ is continuous on $(-\infty, 1) \cup (1, \infty)$.