NAME KEY

Math 12
Test 1
Summer 2011

You have 60 minutes to complete this test. You must *show all work* to receive full credit. Work any 7 of the following 8 problems. Clearly **CROSS OUT** the problem you do not wish me to grade. Each problem is worth 14 points, and you get 2 points for free, for a total of 100 points. If you have any questions, please come to the front and ask.

1. Using the definition of the derivative, find $f'(x)$ if $f(x)=\sqrt{2x+3}$.

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{\sqrt{2(x+h)+3}-\sqrt{2x+3}}{h}\cdot\frac{\sqrt{2(x+h)+3}+\sqrt{2x+3}}{\sqrt{2(x+h)+3}+\sqrt{2x+3}}$$

$$=\lim_{h\to 0}\frac{2(x+h)+3-(2x+3)}{h\left(\sqrt{2(x+h)+3}+\sqrt{2x+3}\right)}=\lim_{h\to 0}\frac{2x+2h+3-2x-3}{h\left(\sqrt{2(x+h)+3}+\sqrt{2x+3}\right)}$$

$$=\lim_{h\to 0}\frac{2h}{h\left(\sqrt{2(x+h)+3}+\sqrt{2x+3}\right)}=\lim_{h\to 0}\frac{2}{\sqrt{2(x+h)+3}+\sqrt{2x+3}}$$

$$=\frac{2}{\sqrt{2x+3}+\sqrt{2x+3}}=\frac{1}{\sqrt{2x+3}}$$

2. Evaluate the following limits. If any of them do not exist, EXPLAIN why not ("because it's undefined" and "denominator is zero" are not sufficient explanations).

(a) $$\lim_{x\to 3}\frac{x+3}{x^2-9}=\lim_{x\to 3}\frac{x+3}{(x+3)(x-3)}=\lim_{x\to 3}\frac{1}{x-3}\quad \underline{\text{DNE}}$$

filling in $x=3$ gives $\frac{1}{0}$, use chart

x	y
2.5	-2
2.9	-10
2.99	-100
3.5	2
3.1	10
3.01	100 ($+\infty$)

(b) $$\lim_{x\to 5}\sqrt[3]{x^2-17}=\sqrt[3]{25-17}=\sqrt[3]{8}=2$$

(c) $$\lim_{x\to 0}\frac{x^2+3x}{x-2x^4}=\lim_{x\to 0}\frac{x(x+3)}{x(1-2x^3)}=\lim_{x\to 0}\frac{x+3}{1-2x^3}=\frac{3}{1}=3$$

3. The quantity x of a particular home office copier is ***inversely proportional*** to the price p. If the price is \$320 each, 240,000 copiers will be sold. How many will be sold if the price is \$480 each?

$X = \frac{K}{P}$, K is the proportionality constant, always the same

$p = 320 \rightarrow x = 240000$, so $240000 = \frac{k}{320}$

$(320)(240000) = k$

$K = 76800000$

$x = \frac{76800000}{P}$

if $p = 480$, then $x = \frac{76800000}{480} = 160{,}000$ copiers will be sold.

4. Find $f'(x)$ (do not simplify!) if :

a) $f(x) = \left(\sqrt[3]{x} - 5x^2 + 4\right)\left(4x^2 + 11x^{-3} - 5\right)$

$f(x) = (x^{1/3} - 5x^2 + 4)(4x^2 + 11x^{-3} - 5)$

$f'(x) = (\frac{1}{3}x^{-2/3} - 10x)(4x^2 + 11x^{-3} - 5) + (x^{1/3} - 5x^2 + 4)(8x - 33x^{-4})$

b) $f(x) = \dfrac{5x^8 - 2x^3}{\left(x^5 - 3\right)\left(x^4 + 7\right)}$

$$f'(x) = \frac{(40x^7 - 6x^2)[(x^5-3)(x^4+7)] - (5x^8 - 2x^3)[(5x^4)(x^4+7) + (x^5-3)(4x^3)]}{[(x^5-3)(x^4+7)]^2}$$

5. Suppose a company can sell x units of a product if the price is set at $p(x) = 50 - 0.5x$, and that the total cost of producing all x units is $C(x) = 4x + 10$.

a) Write an equation to express the revenue from selling x units of the product.

Revenue = price · quantity

$R = (50 - 0.5x)(x) = 50x - 0.5x^2$

b) Write an equation to express the profit from selling x units of the product.

Profit = Revenue − Cost

$P = (50x - 0.5x^2) - (4x + 10) = 46x - 0.5x^2 - 10$

c) What is the ***actual*** profit obtained from the production and sale of the 21st unit?

actual profit $= P(21) - P(20)$

$= [46(21) - 0.5(21)^2 - 10] - [46(20) - 0.5(20)^2 - 10]$

$= 735.5 - 710 = \$25.50$

d) What is the ***marginal*** profit obtained from the production and sale of the 21st unit?

$P'(x) = 46 - x$

$P'(20) = 46 - 20 = \$26$ marginal profit from 21st unit

6. Find the equation of the line tangent to $f(x) = \dfrac{12x^2 - 3x}{3\sqrt{x}}$ at the point where $x = 1$.

point: $x = 1$, $y = \dfrac{12(1)^2 - 3(1)}{3\sqrt{1}} = \dfrac{12 - 3}{3} = 3$ $(1, 3)$

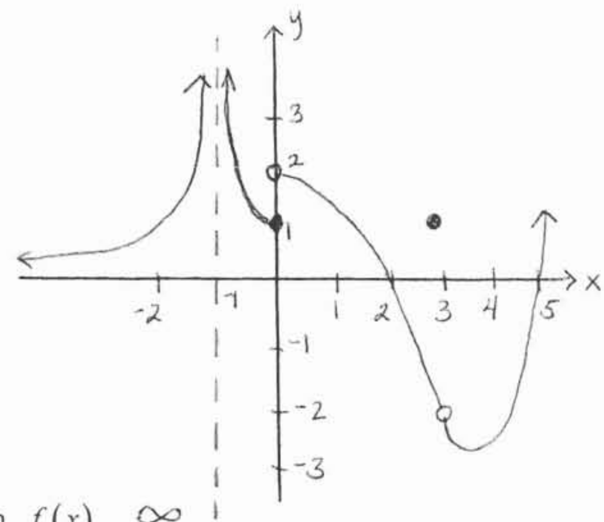
slope: $f'(x) = \dfrac{(24x - 3)(3\sqrt{x}) - (12x^2 - 3x)(\frac{3}{2}x^{-1/2})}{9x}$

$m = f'(1) = \dfrac{(24 - 3)(3) - (12 - 3)(3/2)}{9}$

$= \dfrac{63 - 27/2}{9} = \dfrac{\frac{99}{2}}{9} = \dfrac{11}{2}$

Line: $y - 3 = \frac{11}{2}(x - 1)$

7. Consider the graph of the function $f(x)$ given below.



(a) Find $\lim_{x \to -1} f(x)$. ∞

(b) Find $\lim_{x \to 2} f(x)$. 0

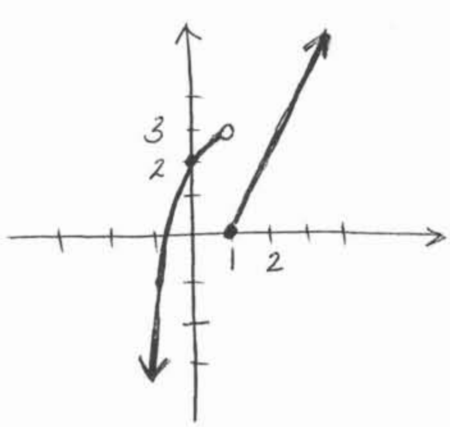
(c) Find $\lim_{x \to 0^-} f(x)$. 1

(d) Find $\lim_{x \to 0^+} f(x)$. 2

(e) Find $\lim_{x \to 0} f(x)$. DNE

(f) Find $\lim_{x \to 3} f(x)$. -2

8. Sketch a graph of the function $f(x) = \begin{cases} -x^2 + 2x + 2 & \text{if} \quad x < 1 \\ 2x - 2 & \text{if} \quad x \geq 1 \end{cases}$. Is this function continuous at $x = 1$? Explain why or why not.



point at $f(1) = 2(1) - 2 = 0$, $(1,0)$

hole if we fill $x = 1$ in to other part
$-1 + 2 + 2 = 3$ hole at $(1,3)$.

The function is NOT continuous at $x=1$, which can be seen from the broken graph. More precisely,

$\left.\begin{array}{l} \lim_{x \to 1^+} f(x) = 1 \\ \lim_{x \to 1^-} f(x) = 3 \end{array}\right\}$ $\lim_{x \to 1} f(x)$ DNE, so not continuous.