NAME_____

You have 50 minutes to complete this test. You must *show all work* to receive full credit. Work any 7 of the following 8 problems. Clearly **CROSS OUT** the problem you do not wish me to grade. Each problem is worth 14 points, and you get 2 points for free, for a total of 100 points. If you have any questions, please come to the front and ask.

1.   Using the definition of the derivative, find $f'(x)$ if $f(x) = 4 + \sqrt{x}$.

2.   Evaluate the following limits. If any of them do not exist, EXPLAIN why not ("because it's undefined" and "denominator is zero" are not sufficient explanations).

(a) $\lim\limits_{x \to 4} \dfrac{4-x}{\sqrt{x}-2}$

(b) $\lim\limits_{x \to 3^+} \dfrac{x+3}{x^2-9}$

(c) $\lim\limits_{x \to 2} \dfrac{x-2}{x^2+3x-1}$

3. Suppose the total cost to produce x snowboards is given by $C(x) = 1000 + 100x - 0.25x^2$ dollars.

a) Using marginal analysis, **estimate** how much the total cost will increase if the production increases from 100 to 102 snowboards.
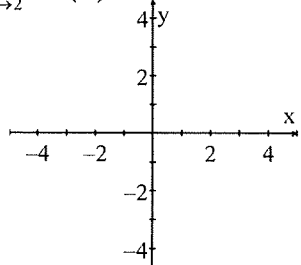
b) Find the **exact** amount of the cost increase.

4. Find $f'(x)$ (do not simplify!) if :
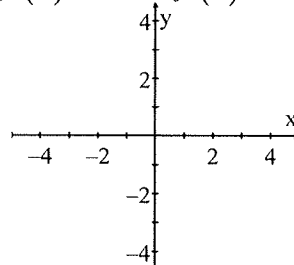
a) $f(x) = \dfrac{2x^2 - 7}{4x + 3}$

b) $f(x) = \left(2x^4 - 3x^3 + x - 5\right)\left(x^2 - x + 5\right)$

5. For each part below, sketch the graph of a function $f(x)$ which satisfies the given condition(s). Although there may be many graphs that will work, only show ONE as your solution.
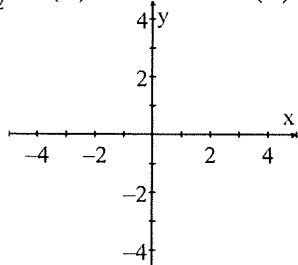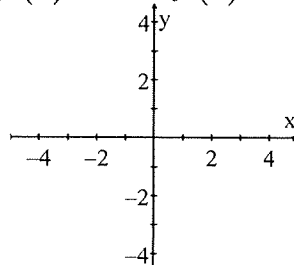
a) $\lim\limits_{x \to 2} f(x) = \infty$

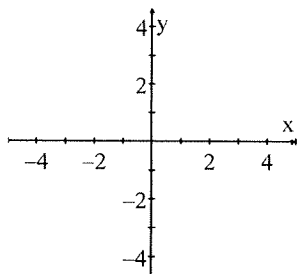d) $\lim\limits_{x \to 2} f(x) = 3$, but $f(2)$ is undefined

b) $\lim\limits_{x \to 2} f(x)$ DNE, but $f(2) = 4$

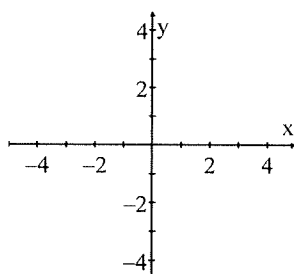e) $\lim\limits_{x \to 2} f(x) = 3$, but $f(2) = 4$

c) $\lim\limits_{x \to 2} f(x)$ DNE, but $\lim\limits_{x \to 2^+} f(x) = 3$

f) $\lim\limits_{x \to 2^+} f(x) = \infty$ and $\lim\limits_{x \to 2^-} f(x) = -\infty$

6. Find the equation of the line tangent to $y = 5\sqrt{x} + 2x^3 - 4x - \dfrac{1}{x^2} + 3$ at the point where $x = 1$.

7. A video production company is planning to produce a set of instructional DVDs. The producer estimates that it will cost $84000 to shoot the video and $15 per set to copy and distribute the DVDs. The wholesale price of the DVDs is $50 per set. Suppose x sets of DVDs are produced.

   a) Write an equation for the total cost function.

   b) Write an equation for the total revenue function.

   c) Find the number of sets that must be produced and sold in order for the company to break even.

8. Sketch a nice big graph of $f(x) = \begin{cases} 1-x^2 & x \le 1 \\ x+2 & x > 1 \end{cases}$. Be sure to clearly label points and axes. Under your graph, list the interval(s) where $f(x)$ is continuous.