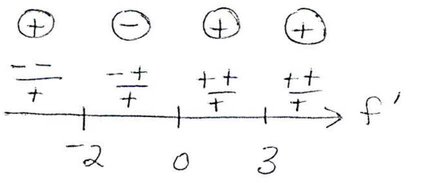
NAME KEY

Math 12
Test 2
Fall 2012

You have 50 minutes to complete this test. You must *show all work* to receive full credit. Work any 7 of the following 8 problems. Clearly **CROSS OUT** the problem you do not wish me to grade. Each problem is worth 14 points, and you get 2 points for free, for a total of 100 points. The answers will be posted on the electronic reserves later today.

1. Suppose that the derivative of $f(x)$ is already calculated to be $f'(x)=\dfrac{3x(x+2)}{(x-3)^2}$. Find the intervals where the original function $f(x)$ is increasing and where it is decreasing (interval notation, please). List the critical numbers and state whether each gives a relative maximum, minimum, or neither.

CN: $x = 0, -2, 3$



increasing on $(-\infty,-2)\cup(0,3)\cup(3,\infty)$
decreasing on $(-2,0)$

$x=-2$ gives a max
$x=0$ gives a min
$x=3$ gives neither

2. For the following functions, find all horizontal and vertical asymptotes (remember that an asymptote is a LINE, not a number). If there are no asymptotes, say so.

a) $f(x)=\dfrac{1}{4-x^2} = \dfrac{1}{(2-x)(2+x)}$

VA: $x=\pm 2$
HA: $y=0$

b) $f(x)=\dfrac{4x^2}{x^2+1}$

$x^2+1=0$
$x^2=-1$ No.

VA: none
HA: $y=4$

c) $f(x)=\dfrac{x^3+2x^2-3x}{x^2+x-6} = \dfrac{x(x+3)(x-1)}{(x+3)(x-2)}$

VA: $x=2$
($x=-3$ gives a hole)
HA: none

3. Suppose that at price p, demand for a certain product is given by $q(p)=\dfrac{500-2p}{p}$ when price is a positive value less than \$250.

a) Find the price elasticity of demand when price is \$50. Is demand elastic or inelastic at this price?

$$E(p) = \frac{p}{q}\cdot q' = p\cdot\frac{p}{500-2p}\cdot\frac{-2p-(500-2p)}{p^2} = \frac{-500}{500-2p}$$

$$E(50) = \frac{-500}{500-100} = \frac{-5}{4}.$$

$\left|\frac{-5}{4}\right| > 1$, so elastic.

b) Complete this statement:

According to my answer in (a), if the price of \$50 goes ***up/down*** (circle one) by 1%, the demand for the product will go ***up/down*** (circle one) by 1.25%.

c) Give an example of a product in the correct price range that might behave as described in (a) and (b).

Any luxury item around \$50 in price.
(video game, nice dinner...)

4. Describe the concavity and find the inflection points of $f(x)=\dfrac{4x^2}{x^2+1}$. Do not sketch the graph.

$$f'(x) = \frac{8x(x^2+1)-4x^2(2x)}{(x^2+1)^2} = \frac{8x^3+8x-8x^3}{(x^2+1)^2} = \frac{8x}{(x^2+1)^2}$$

$$f''(x) = \frac{8(x^2+1)^2-8x(2)(x^2+1)(2x)}{(x^2+1)^4} = \frac{8(x^2+1)-32x^2}{(x^2+1)^3}$$

$$= \frac{-24x^2+8}{(x^2+1)^3} = \frac{-8(3x^2-1)}{(x^2+1)^3} = 0$$

$3x^2-1=0$
$x^2 = 1/3$
$x = \pm 1/3 \rightarrow y = \dfrac{4/3}{\frac{1}{3}+1} = 1$

$\ominus \quad \oplus \quad \ominus \quad \rightarrow f''$

$\frac{-1}{\sqrt{3}} \quad \frac{1}{\sqrt{3}}$

Concave up on $\left(\frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

concave down on $\left(-\infty, \frac{-1}{\sqrt{3}}\right) \cup \left(\frac{1}{\sqrt{3}}, \infty\right)$

inflection points $\left(\pm\frac{1}{\sqrt{3}}, 1\right)$

5. Find all absolute extrema of $f(x) = -3x^5 + 5x^3$ on the interval $[-2, 0]$.

$f'(x) = -15x^4 + 15x^2 = -15x^2(x^2 - 1) = -15x^2(x+1)(x-1)$

CN: $x = 0, -1, 1 \leftarrow$ not in interval

$f(0) = 0$

$f(-1) = 3 - 5 = -2$

$f(-2) = -3(-32) - 40 = 56$

↑ check end point, too!

abs max $(-2, 56)$

abs min $(-1, -2)$

6. Sketch the graph of a function $f(x)$ so that all conditions below are satisfied. Be sure your graph is big enough so I can see it and it is properly labeled.

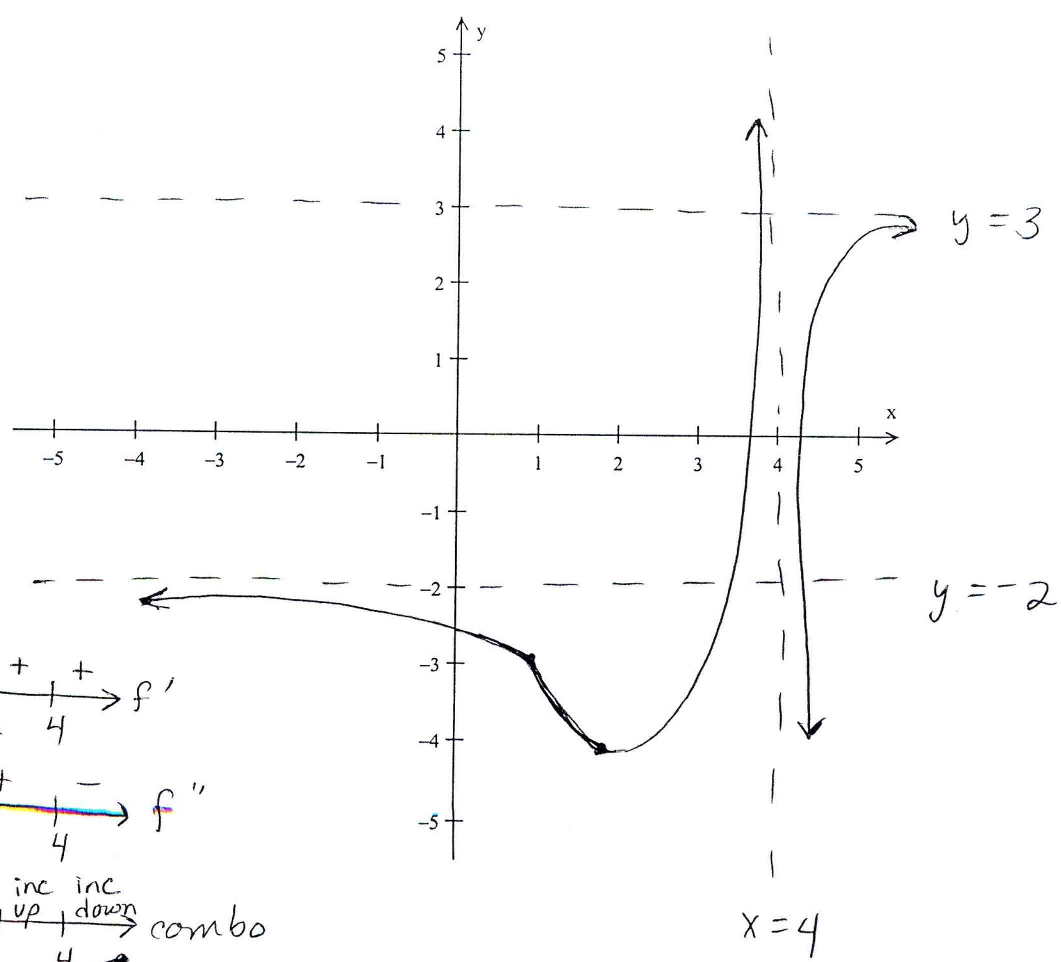
a) $\lim_{x\to\infty} f(x) = 3$, $\lim_{x\to-\infty} f(x) = -2$, and $f(x)$ is defined for all $x \neq 4$.

b) $f'(x) < 0$ when $x < 2$, but $f'(x) > 0$ when $2 < x < 4$ and when $x > 4$.

c) $f''(x) < 0$ when $x < 1$ and when $x > 4$, but $f''(x) > 0$ when $1 < x < 4$.

horiz asymp
$y = 3$
$y = -2$

vert asymp
$x = 4$
(or a hole)



f': $-$ | 2 | $+$ | 4 | $+$

f'': $-$ | 1 | $+$ | 4 | $-$

combo: dec down | 1 | dec up | 2 | inc up | 4 | inc down

7. Find the equation of the line tangent to $3x^2y^3 - x + y = 25$ at the point (1,2).

$$6xy^3 + 3x^2 3y^2 y' - 1 + y' = 0$$

$x = 1$, $y = 2$, so

$$6(1)(8) + 3(1)(3)(4)y' - 1 + y' = 0$$

$$48 + 36y' - 1 + y' = 0$$

$$37y' = -47$$

$$y' = -47/37 = m$$

line: $y - 2 = \frac{-47}{37}(x - 1)$

8. Your company has received an order to make 400,000 customized ball point pens. You own 20 machines, each of which can produce 200 pens per hour. The cost of setting up the machines to produce the pens is \$80 per machine, and the total operating cost is \$5.76 per hour. How many machines should be used to minimize the cost of producing the 400,000 pens? What is the minimum cost?

Cost = setup cost + operating cost

C = 80(# machines) + 5.76 (# hours)

If x = # machines, we will produce 200x pens/hr.
To make 400,000 pens, it will take

$$400{,}000 \text{ pens} \times \frac{1 \text{ hour}}{200x \text{ pens}} = \frac{20000}{x} \text{ hours.}$$

$$C = 80x + 5.76(20000x^{-1}) = 80x + 115200x^{-1}$$

$$C' = 80 - \frac{115200}{x^2} = 0, \quad 80x^2 = 115200$$

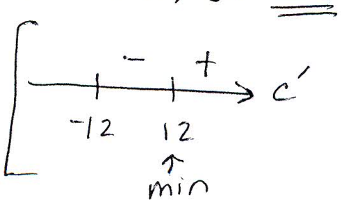
$$x^2 = 144$$

$$x = \pm 12$$

$\left[\begin{array}{l} C'' = 2(115200)x^3 \\ C''(12) > 0, \text{ so } \underline{\text{min}} \end{array}\right.$

$x = 12$ machines will minimize cost.

OR



min cost is

$$C(12) = 80(12) + 5.76\left(\frac{20000}{12}\right)$$

$$= \boxed{\$10560}$$